SOME EXTENSIONS OF THE ROOT LOCUS METHOD

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by

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ABSTRACT

Simple analytical methods for the determination of the angle of emergence from complex poles and zeros are developed. Several case studies of root loci emerging from complex zeros are analyzed to determine criterion for the loci starting from complex pole to end on complex zero, as a pre-requisite study for "cancel-compensation" and self-adaptive compensators.

The writer wishes to thank Dr. George J. Thaler for his advice and encouragement in preparing this paper.

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α - angle defined in Fig. 6 and equations (2-16) and (2-17)

β - constant angle defined by equation (2-7) and (2-8)

η - angle defined in Fig. 6 and equations (2-16) and (2-17)

θ - angular position of complex zero with respect to its neighboring complex pole

$\angle K_p$ - residue angle of complex pole of "uncompensated" system (no complex zeros)

$\angle K_{pc}$ - residue angle of complex pole with adjacent "compensating" zeros

$\angle K_z$ - residue angle of complex zero

λ - angle defined in Fig. 4

ϕ - angle defined in Fig. 5

ψ_c - angle of emergence from a complex pole with adjacent "compensating" complex zero

ψ_p - angle of emergence from a complex pole ("uncompensated system")

ψ_z - angle of emergence from a complex zero

CHAPTER I

INTRODUCTION

Many closed loop transfer functions of control systems such as those representing structual resonances in missiles, fuel sloshing, shaft twisting, etc., have a pair of complex conjugate poles near the imaginary axis. If gain requirements are not too high, the effect of the roots near these poles is often neglected because other roots dominate. However, in the design of high gain systems, these roots near the imaginary axis may cross over into the right half s-plane and thus cause instability in the system. Figure 1 shows the effect of raising gain on the open-loop Bode diagram.

Cancel compensation would be an effective solution to removing these complex poles near the jw-axis if it were not for (1) the difficulty in the design accuracy of these compensators, and (2) the poles themselves may not be fixed, and may move about in an area in the s-plane.

The mere placing of complex zeros near the complex poles is no assurance that the root loci is connected between these poles and zeros. (See Fig. 2) When this loci does close on the adjacent complex zero, the root on this locus has such a small residue that it can be neglected when compared with the other roots.

The question remains: What are the requirements for the location of complex zeros adjacent to complex poles for best cancel compensation? The search for an answer to this question initiated this study. Such

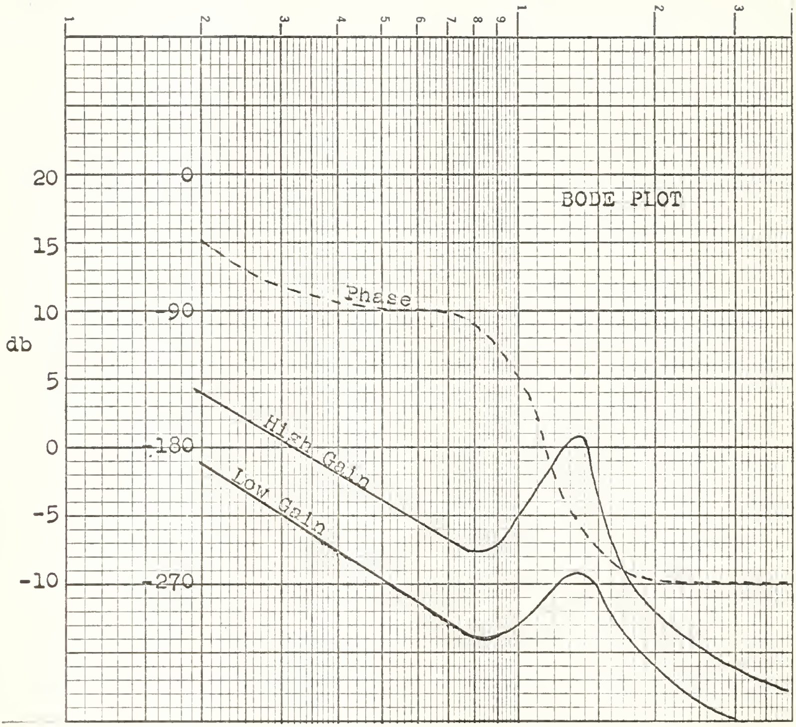


Fig. 1

Effect of a pair of Complex Poles near the Imaginary Axis on High Gain Systems

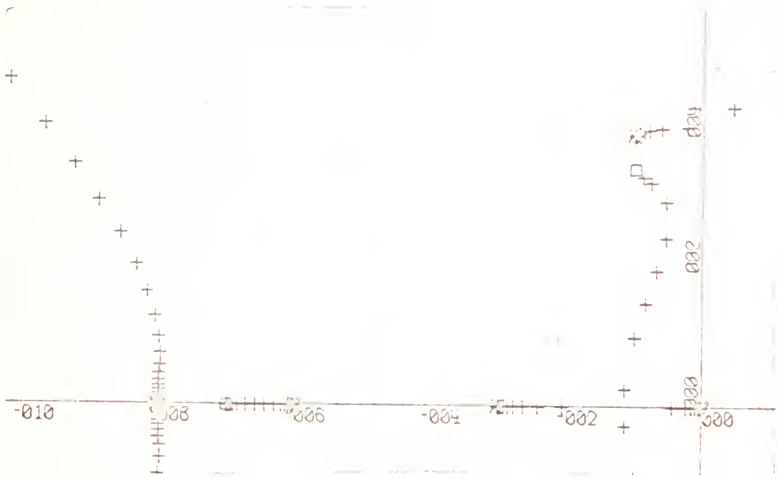


Fig. 2

Root Loci from Complex Pole with adjacent Complex Zero a location or area might make it possible to locate complex zeros for the design of self-adaptive compensators.

First in this study, the angle of emergence from a complex pole and zero was studied to determine how it varied and how it was related to the transfer function.

Next, 43 different case studies were programmed into the Control Data Corporation 1604 Computer to obtain 215 root locus plots. These plots and observations are contained in Chapter 3.

The term "compensated system" will be defined hereafter to mean a system with a pair of complex conjugate zeros relatively close to complex conjugate poles. The "uncompensated system" will be defined as the system without the complex conjugate zeros, but will include the real poles associated with a "zero-compensator."

CHAPTER II

ANGLE OF EMERGENCE

1. Determination of the Angle of Emergence

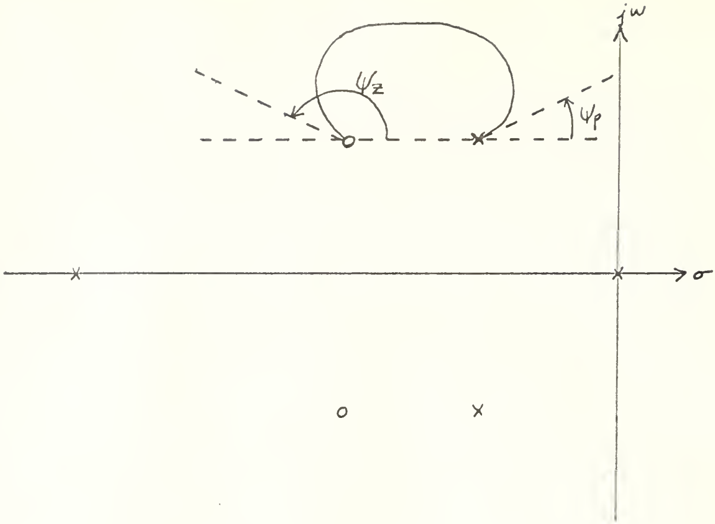
The angle of emergence of the root locus from a complex pole or zero has been defined in the literature as the angle of the tangent to the root-locus curve at the pole or zero as shown in Fig. 3. A test point s_1 near the complex pole on the root locus satisfies the equation

$$\sum_{i=1}^{N} \angle Z_i - \sum_{i=2}^{M} \angle P_i - \angle P_1 = 180 + n(180)$$

where n is an odd integer or zero, N is the total number of zeros, M is the total number of poles, and $\angle P_1$ is the angle of the complex pole infinitesimally close to the test point. Fig. 3 shows that $\angle P_1$ is measured counter-clockwise with the negative abscissa axis as reference, whereas the angle of emergence, ψ_p is measured counterclockwise with the positive abscissa axis as reference. It is thus seen that the angle of emergence is equal to the angle from the test point to the complex pole, using their corresponding references. Thus,

$$\psi_p = \angle P_1 = 180 + \sum_{i=1}^{N} \angle Z_i - \sum_{i=2}^{M} \angle P_i \qquad (2\text{-}1)$$

The last two terms in the above equation can be determined with the complex pole or zero (from which the angle of emergence is to be determined) as the origin of the coordinate system, since the test point s_1 is infinitesimally close to the complex pole (or zero). Thus, the origin of the s-plane is translated to the desired complex pole or zero by setting $w = s + \alpha - j\beta$ where w is the new complex variable in



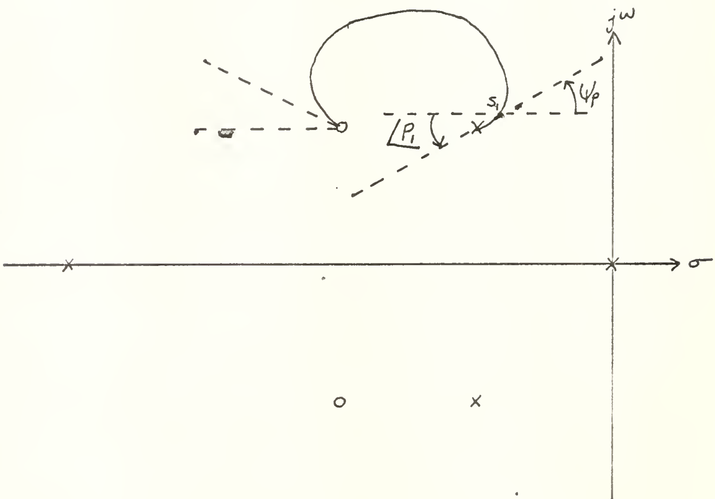


Fig. 3

Angle of Emergence of complex pole, ψ_p
Angle of Emergence of complex zero, ψ_z
Angle from test point s_1 to complex pole, $\angle P_1$

the w-plane. A test point w_1 near the new origin (pole or zero) which is on the root-locus must still satisfy the angle criterion (180 degrees or an odd multiple). As $|w|$ approaches zero the sum of the angles on the root locus plus 180 degrees would be the angle from the origin to the test point. When $|w|$ is identically zero, there is no angular contribution from the pole or zero at the origin and thus the sum of the remaining angles is the angle of emergence. For example, a characteristic equation is partitioned as follows:

$$\frac{(s + 3 - j5)(s+3+j5)}{s(s+10)(s+2-j5)(s+2+j5)} = -1$$

To find the angle of emergence at the complex pole located at $-2 + j5$ in the s-plane, transfer the origin to this pole by setting $w = s+2 - j5$ or $s = w - 2 + j5$ Then,

$$\frac{(w-2+j5+3-j5)(w-2+j5+3+j5)}{(w-2+j5)(w-2+j5+10)(w-2+j5+2-j5)(w-2+j5+2+j5)} = -1$$

$$\frac{(w+1)(w+1+j10)}{(w-2+j5)(w+8+j5)(w)(w+j10)} = -1$$

$$w = \frac{(w+1)(w+1+j10)}{(-1)(w-2+j5)(w+j10)(w+8+j5)} \qquad (2\text{-}2)$$

Define the angle of emergence from the complex pole,

$$\psi_p \equiv \angle w \,\Big|_{|w| = 0} \qquad (2\text{-}2a)$$

$$\psi_p = \tan^{-1}\frac{0}{1} + \tan^{-1}\frac{10}{1} - \left(180 + \tan^{-1}\frac{5}{-2} + \tan^{-1}\frac{5}{8} + 90\right)$$

$$\psi_p = 0 + 84.5 - (180 + 111.6 + 32 + 90)$$

$$\psi_p = 30.9$$

Now to find the angle of emergence from the complex zero located at $-3 + j5$ in the s-plane, transfer the origin to this zero by setting $w = s + 3 - j5$ or $s = w - 3 + j5$

$$\frac{(w-3+j5+3-j5)(w-3+j5+3+j5)}{(w-3+j5)(w-3+j5+10)(w-3+j5+2-j5)(w-3+j5+2+j5)} = -1$$

$$\frac{(w)(w+j10)}{(w-3+j5)(w+7+j5)(w-1)(w-1+j10)} = -1$$

$$w = \frac{(-1)\ (w-3+j5)(w+7+j5)(w-1)(w-1+j10)}{(w+j10)} \tag{2-3}$$

Define the angle of emergence from the complex zero,

$$\psi_z = \angle w \Big|_{|w| = 0} \tag{2-3a}$$

$$\psi_z = 180 + \tan^{-1}\frac{5}{-3} + \tan^{-1}\frac{5}{7} + \tan^{-1}\frac{0}{-1} + \tan^{-1}\frac{10}{-1}$$

$$- 90$$

$$\psi_z = 180 + 121 + 35.5 + 180 + 95 - 90$$

$$\psi_z = 161.5$$

2. Complex Residues and the Angle of Emergence

The closed loop transfer function can be expanded by partial fraction expansion and the residues evaluated by use of Heaviside's expansion theorem: (a) multiply function by $(s - p_k)$, and (b) let $s = p_k$, giving the residue at K_{pk}. Using the previous example again, the residue angle at $s = -2 + j5$ is:

$$\angle K_{rc} = \frac{(s+3-j5)(s+3+j5)}{s(s+10)(s+2+j5)} \Bigg|_{s = -2+j5} \tag{2-4}$$

It is seen that this residue evaluation is similar to equation (2-1) except for the -1 in the denominator which introduces 180 degrees. Thus the angle of the residue at a complex pole differs from the angle of emergence by 180 degrees:

$$\psi_{pc} = \angle K_{rc} + 180 \tag{2-5}$$

The characteristic equation of the preceding section may be partitioned as follows, where the zeros are now the "poles" and the poles are now the "zeros":

$$\frac{s(s+10)(s+2-j5)(s+2+j5)}{(s+3-j5)(s+3+j5)} = -1$$

The left side of this equation is seen to be the reciprocal of the closed-loop transfer function. Although this is an improper fraction, the residues can still be evaluated by Heaviside's expansion theorem. Thus, the residue at $s = -3+j5$ is:

$$\angle K_z = \frac{s(s+10)(s+2-j5)(s+2+j5)}{(s+3+j5)} \qquad s = -3+j5 \tag{2-6}$$

$$\angle K_z = \frac{(-3+j5)(-3+j5+10)(-3+j5-2-j5)(-3+j5+2+j5)}{(-3+j5+3+j5)} \tag{2-7}$$

$$\angle K_z = 341.5$$

Comparing equations (2-7) and (2-3) it is seen that they differ by 180 degrees. Thus the angle of emergence from a complex zero is

$$\psi_z = \angle K_z + 180 \tag{2-7a}$$

where $\angle K_z$ is determined as illustrated above.

3. Effect of Complex Zeros near Complex Poles

What effect does the relative position of complex zeros adjacent to complex poles have on the angle of emergence from the complex poles? To answer this question, first consider the change of the angle of emergence with the zeros at constant radius from the poles while varying the angular position of the zeros with respect to the complex poles.

It is assumed that the displacement between complex poles and the distance between complex pole and its neighboring zero is of a ratio of approximately 10:1. As seen in Fig. 4, as the angle θ, the angular position of the

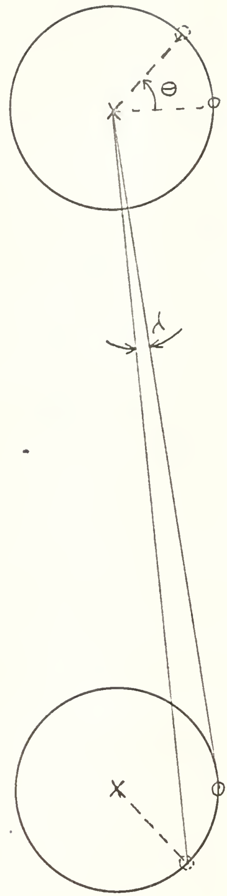


Fig. 4

Effect of Zero Angular Position on Angle of Emergence complex zero, increases, the angle of emergence increases in magnitude. The angular contribution of λ due to the lower half plane zero is negligible compared with θ in the upper half plane. Therefore a linear slope of one exists between angular position of zero and angle of emergence, or

$$\frac{\partial \psi}{\partial \theta} \cong 1 \tag{2-8}$$

Next consider Fig. 5 which shows the effect of radial position of complex zeros (with constant angular position) on the angle of emergence. As the zeros vary in radial position, there is no angular change due to upper half plane zero. The change in lower half plane zero angle is ϕ. In addition to the assumption stated above, the line P_1Z_2 is assumed perpendicular to line P_2A for small changes in radial position r. Then

$$\phi = \tan^{-1}\left(\frac{r\cos\theta}{d - r\sin\theta}\right)$$ where d is the displacement distance between complex poles, P_1P_2

$$\frac{\partial \phi}{\partial r} = \frac{\frac{\partial}{\partial r}\left(\frac{r\cos\theta}{d - r\sin\theta}\right)}{1 + \frac{r^2\cos^2\theta}{(d - r\sin\theta)^2}}$$

$$\frac{\partial \phi}{\partial r} = \frac{\left[(r\cos\theta)(-1)(d-r\sin\theta)^{-2}(-\sin\theta) + (d-r\sin\theta)^{-1}\cos\theta\right]}{1 + \frac{r^2\cos^2\theta}{(d - r\sin\theta)^2}}$$

$$\frac{\partial \phi}{\partial r} = \frac{\left[\frac{r\sin\theta\cos\theta}{(d - r\sin\theta)^2} + \frac{\cos\theta}{d - r\sin\theta}\right]}{1 + \frac{r^2\cos^2\theta}{(d - r\sin\theta)^2}}$$

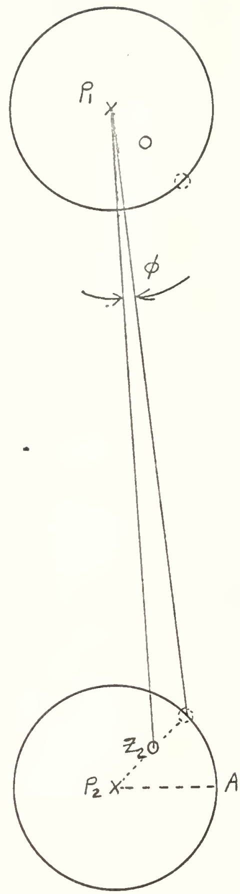


Fig. 5

Effect of Zero Radial Displacement on Angle of Emergence

$$\frac{\partial\phi}{\partial r} = \frac{r \sin\theta \cos\theta \;+\; \cos\theta(d - r\sin\theta)}{(d - r\sin\theta)^2 \;+\; r^2\cos^2\theta}$$

$$\frac{\partial\phi}{\partial r} = \frac{d \cos\theta}{d^2 - 2dr\sin\theta + r^2}$$

Sine it was assumed that $d/r = 10$, $d^2 \gg -2dr\sin\theta + r^2$

$$\frac{\partial\phi}{\partial r} = \frac{\cos\theta}{d} = \tag{2-9}$$

The total derivative of the angle of emergence is:

$$d\psi = \frac{\partial\psi}{\partial\theta}\,d\theta + \frac{\partial\psi}{\partial r}\,dr$$

For small displacements,

$$\psi_c = \theta + \frac{r\cos\theta}{d} + \beta \tag{2-10}$$

where ψ_c is the angle of emergence of the compensated system, and β is a constant. If r, the distance between adjacent complex poles and zeros, is allowed to become zero, or "perfect cancellation" with poles, the angular position of the zeros is undefined. The constant β is then evaluated as the sum of all remaining zeros in the system minus the sum of all remaining poles, measured from the upper half plane "cancelled" complex pole. Thus in the example cited in section 1, β is evaluated as 210.9. For θ = 180, the angle of emergence ψ_p = 30.9 degrees. The second term on the right side of equation (2-10) is negligible compared with the other terms under the assumptions that have been made. (See Fig. 7) It is seen that the angle β differs from the residue angle of the uncompensated system (no complex zeros present) by 90 degrees.

$$\angle K_p = \beta - 90 \tag{2-11}$$

The angle of emergence of the uncompensated system, ψ_p, is readily determined:

$$\psi_p = K_p + 180 \tag{2-5a}$$

$$\psi_p = \beta - 90 + 180 = \beta + 90 \tag{2-12}$$

Equation (2-7) can now be written as:

$$\psi_c = \theta + \frac{\cos\theta}{d} r + \psi_p - 90$$

$$\psi_c \cong \theta + \psi_p - 90 \tag{2-13}$$

Thus the angle of emergence changes directly with the zero position angle θ. (See Fig. 8)

The residue angles of the complex pole and zero are defined in equations (2-4) and (2-7) respectively and can be written in the following form:

$$\angle K_{pc} = \left.\frac{\angle z_1 \angle z_2}{\angle p_1 \angle p_2 \angle p_3}\right|_{\text{at pole } p_4} \tag{2-14}$$

$$\angle K_z = \left.\frac{\angle p_1 \angle p_2 \angle p_3 \angle p_4}{\angle z_1}\right|_{\text{at zero } z_2} \tag{2-15}$$

The relationship between the pole residue angle, K_{pc} and the zero residue angle K_z can now be written as:

$$\angle K_z = -\angle K_{pc} + \angle z_2 + \angle p_4 \tag{2-16}$$

Let $\alpha = \angle z_2$ and $\eta = \angle p_4$ The equation (2-16) becomes:

$$\angle K_z = -\angle K_{pc} + \alpha + \eta \tag{2-17}$$

The angles α and η are seen in Fig.6. It is observed from this figure that $\alpha = \theta - 180$ and $\eta = \theta$

Substituting these relationships into equation (2-17) yields the following result:

$$\angle K_z = -\angle K_{pc} + 2\theta - 180 \tag{2-18}$$

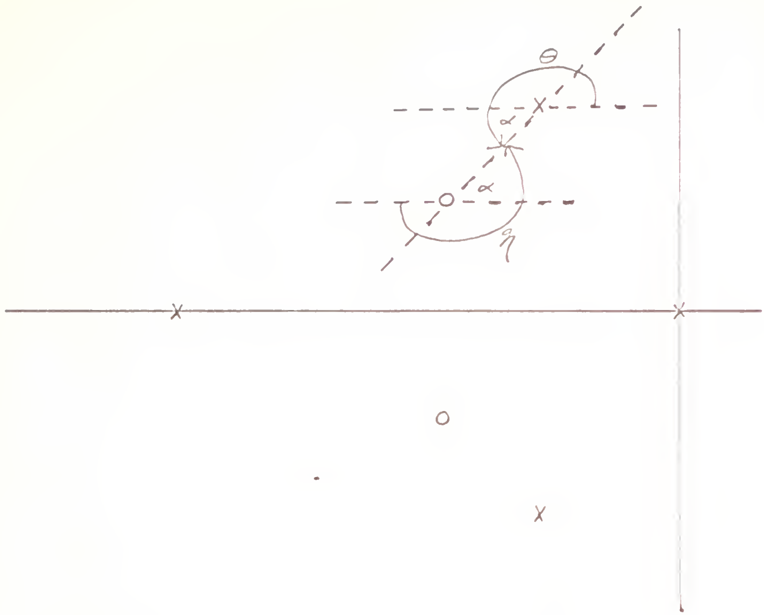


Fig. 6

Relationship between Residue Angle of Complex Pole and Complex Zero.

$$\angle K_z = \angle K_p + \alpha + \eta$$

But from equations (2-5) and (2-7a),

$$\angle K_{pc} = \psi_c - 180$$

$$\angle K_z = \psi_z - 180$$

Therefore,

$$\psi_z - 180 = -\psi_c + 180 + 2\theta - 180 \qquad \text{or,}$$

$$\psi_z = -\psi_c + 2\theta + 180 \tag{2-19}$$

Substituting equation (2-13) into equation (2-19) yields the relation between the angle of emergence from the complex zero and the angle of emergence from the complex pole when the complex zeros are not present.

$$\psi_z = -\theta - \psi_p + 90 + 2\theta + 180$$

$$\psi_z = \theta - \psi_p - 90 \tag{2-20}$$

Using the relation from equation (2-5a), then

$$\psi_z = \theta - \angle K_p + 90 \tag{2-21}$$

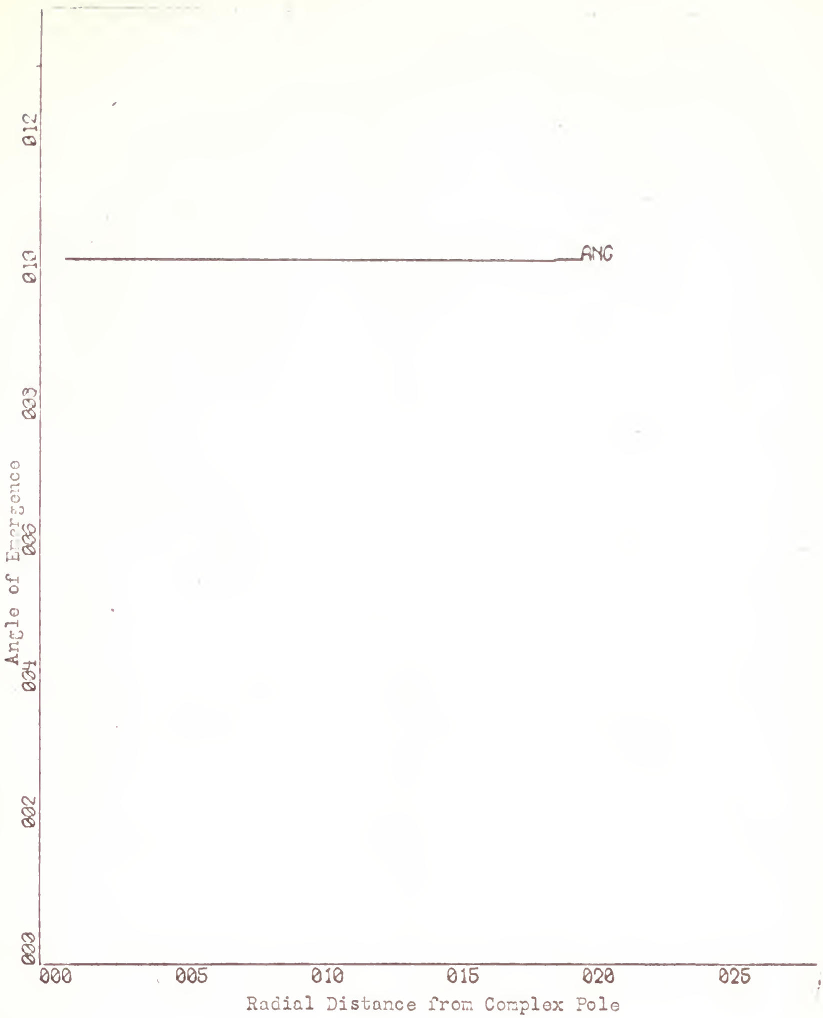
Subtracting equation (2-13) from equation (2-20) gives the relation:

$$\psi_z - \psi_c = -2\psi_p = -2(\angle K_p + 180)$$

$$\psi_z - \psi_c = -2\angle K_p \tag{2-22}$$

Thus the difference between the angle of emergence from a complex pole and the angle of emergence of its neighboring complex zero is a constant for a given system, and thus not a function of zero position θ.

Figures 7 and 8 are computer-solution graphs of the angle of emergence versus complex zero radial and angular positions respectively.



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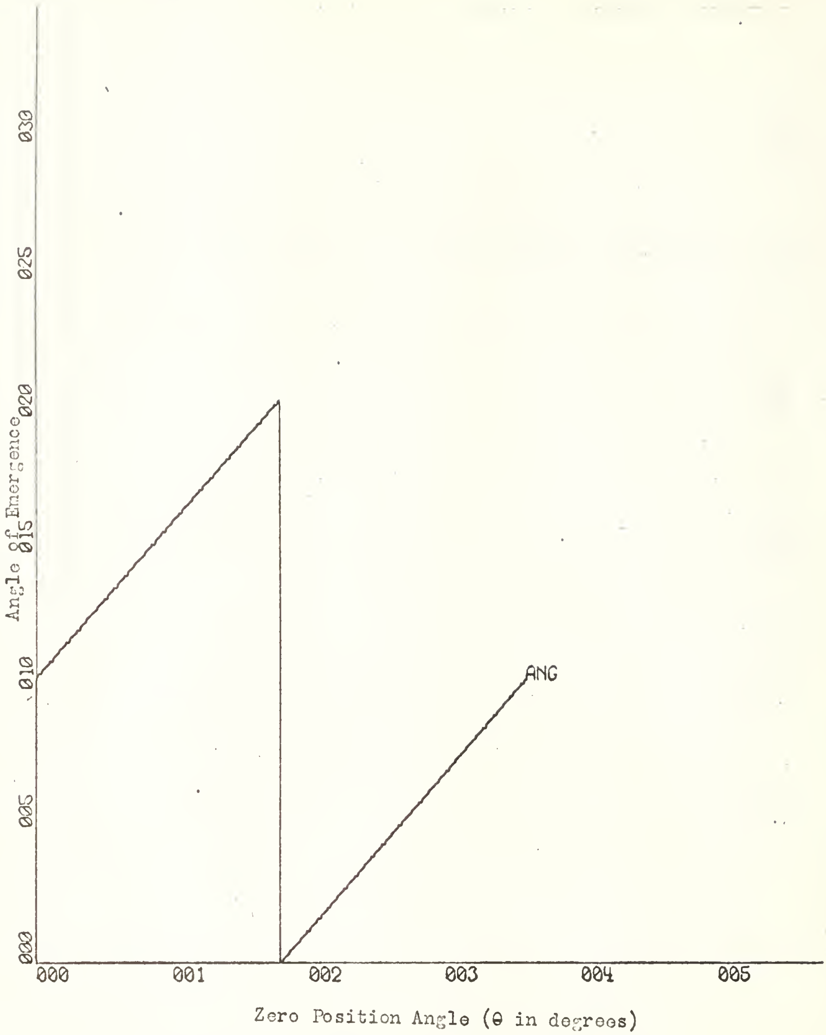
Y-SCALE - 2.00E+01 UNITS/INCH

Fig. 7

Angle of Emergence vs. Radial Position of Complex Zero (θ constant)



X-SCALE - 1.00E+02
Y-SCALE - 5.00E+01

Fig. 8

Angle of Emergence vs. Zero Position Angle, θ

CHAPTER III

ROOT LOCI LEAVING COMPLEX POLES

1. Data Designation

Having analyzed the angles of emergence from complex poles and zeros, the next step in this study is to determine the effect of various system transfer functions on the root loci emerging from the complex poles, and to determine when this loci closes on its adjacent complex zero, i.e., the root loci starts at the complex pole and ends on the complex zero. As shown in Fig.2 of Chapter I, this loci may go to the zero at infinity instead of the complex zero.

Over fifty different cases were tried and the data consisted of about 300 root locus plots using the computer program in Appendix I. Only 43 case studies (Fig. 9) are presented with 215 individual root locus plots (Fig. 10) for this analysis.

Each case study or individual root locus plot is designated or classified using letters and numbers, such as "A-1a", where the first (capital) letter designates generally the number of real poles and zeros, and position of complex conjugate poles. The number indicates the variable position of the given number of real poles and zeros. The last small letter specifies the variable position of the complex zero with respect to the complex pole.

Double, triple, etc., real poles are specified by placing an "X" over an "X" or a group of "X's" near appropriate point on the plot.

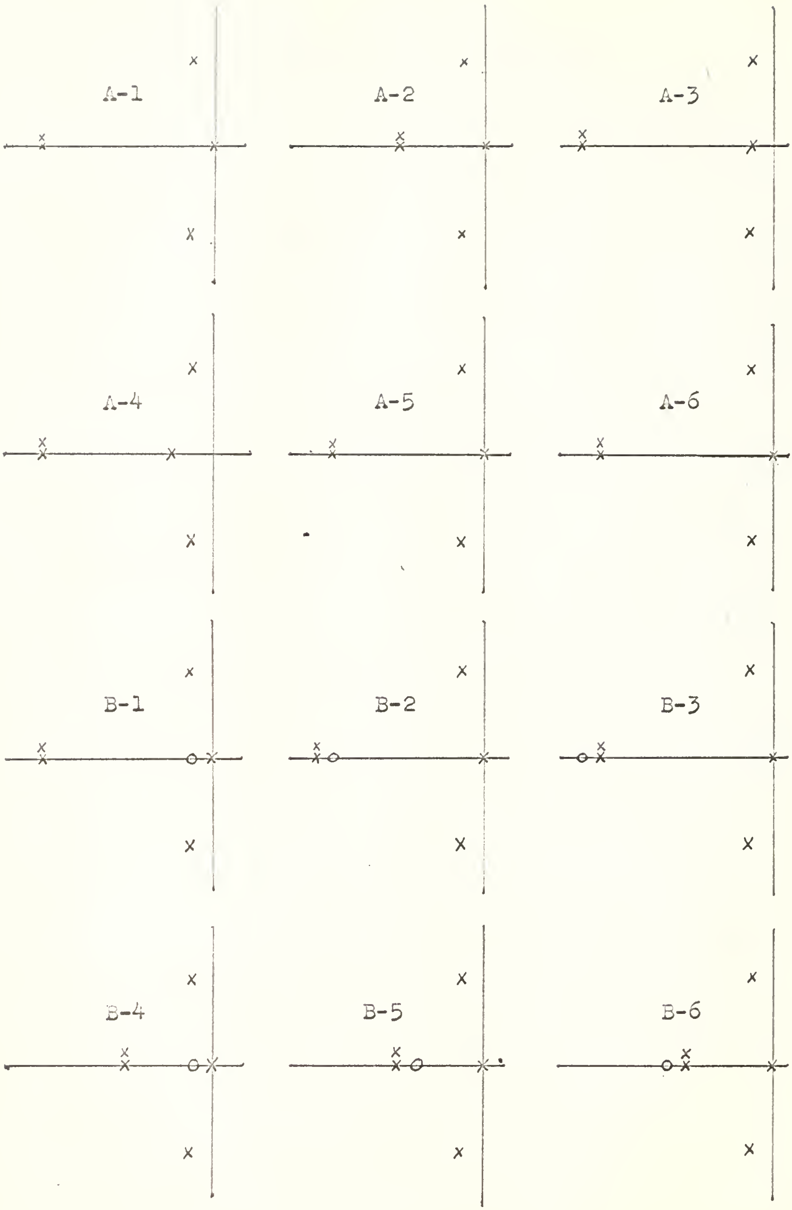


Fig. 9a Case studies

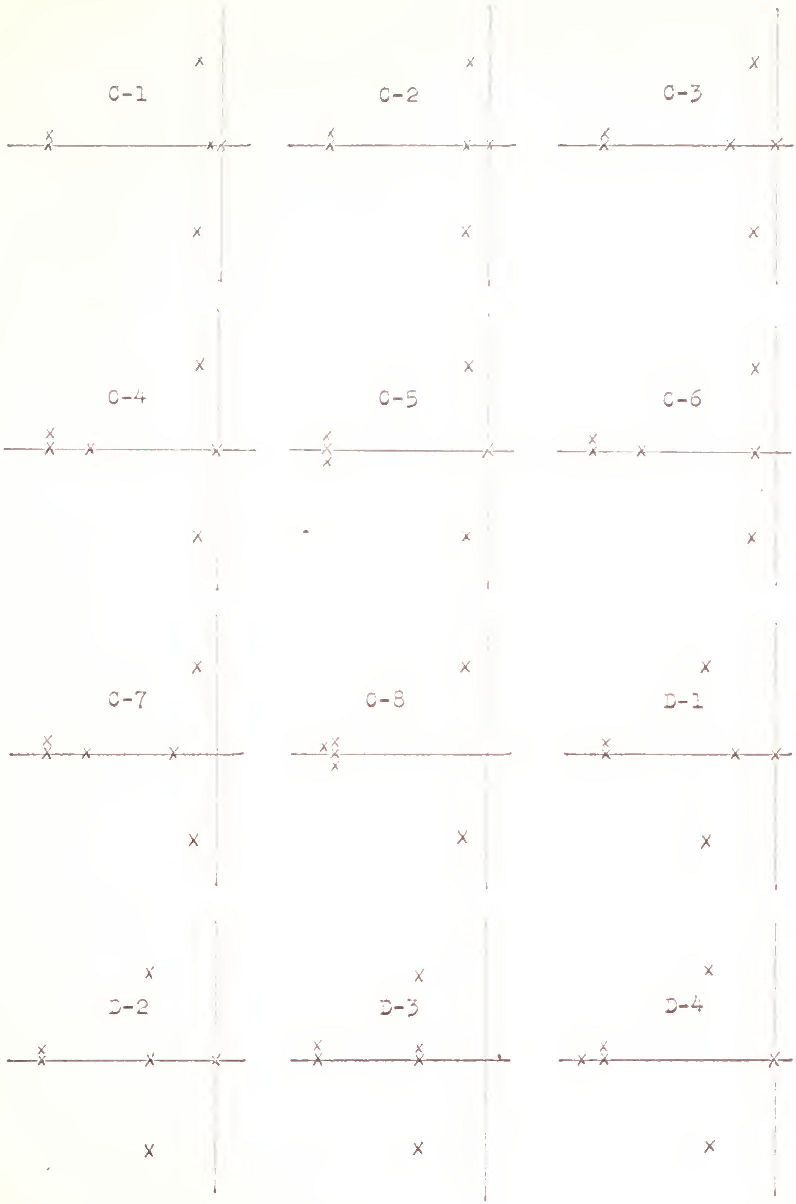


Fig. 9b Case studies

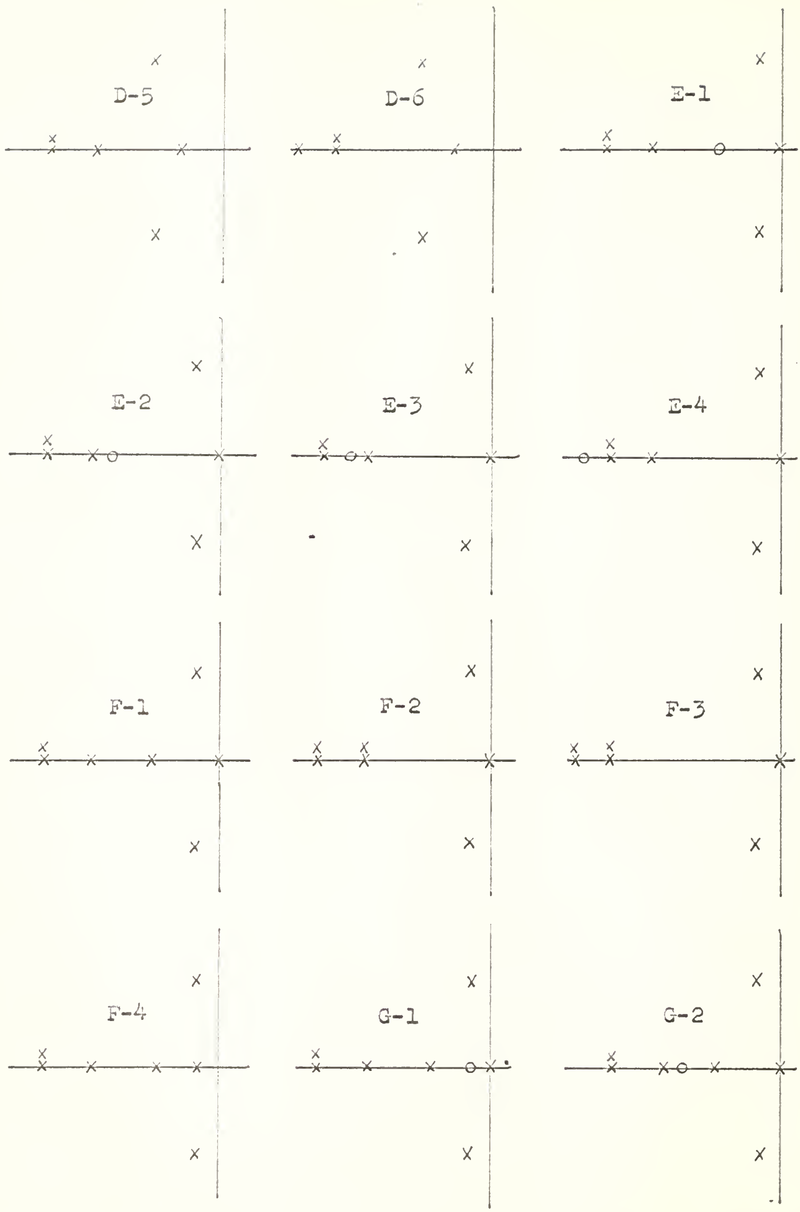


Fig. 9c Case studies

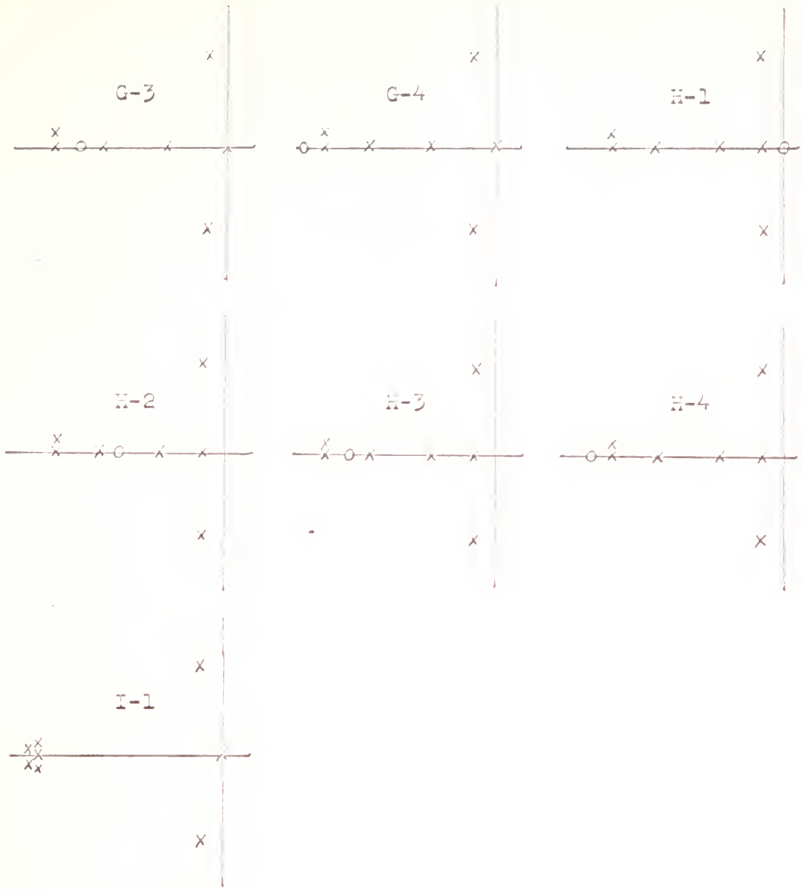


Fig. 9d Case studies

The double real poles are arbitrarily used to represent the poles associated with a "cancel-compensator" and is usually located, for this study, at -8 units except in a few cases they are placed at -4 units.

Case A is a fifth order system. Case B is a fifth order system with one real zero. Case C is a sixth order system. Case D is a sixth order system with the complex conjugate poles moved out to $-3 \pm j4$. Case E is a sixth order system with one real zero. Case F is a seventh order system. Case G is a seventh order system with one real zero. Case H is also a seventh order system with one real zero. Case I is an eighth order system. Figure 9 presents these 43 case studies, and shows only the pole-zero configurations without the complex conjugate zeros. Figure 10 shows the root locus plots for these case studies when the complex zeros are placed in various positions.

Table 1 is a summary of the graphical data from Fig. 10. In addition it contains the computed values of angles of emergence from pole and zero (for $\theta = 0°$) and the uncompensated residue angle using the formulas developed in Chapter II. A comparison with these calculated values from the transfer function compares very good with computer root locus plots.

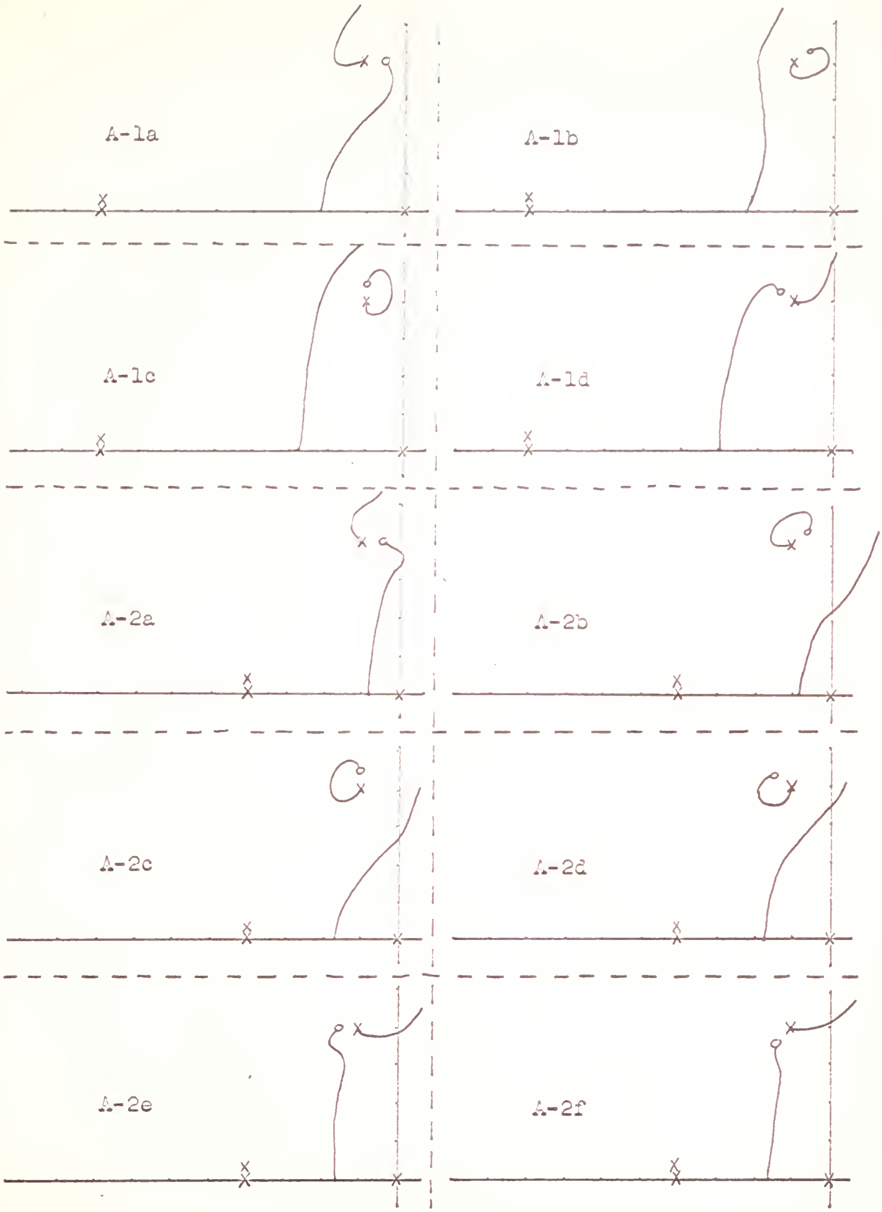


Fig. 10a Root Locus Data

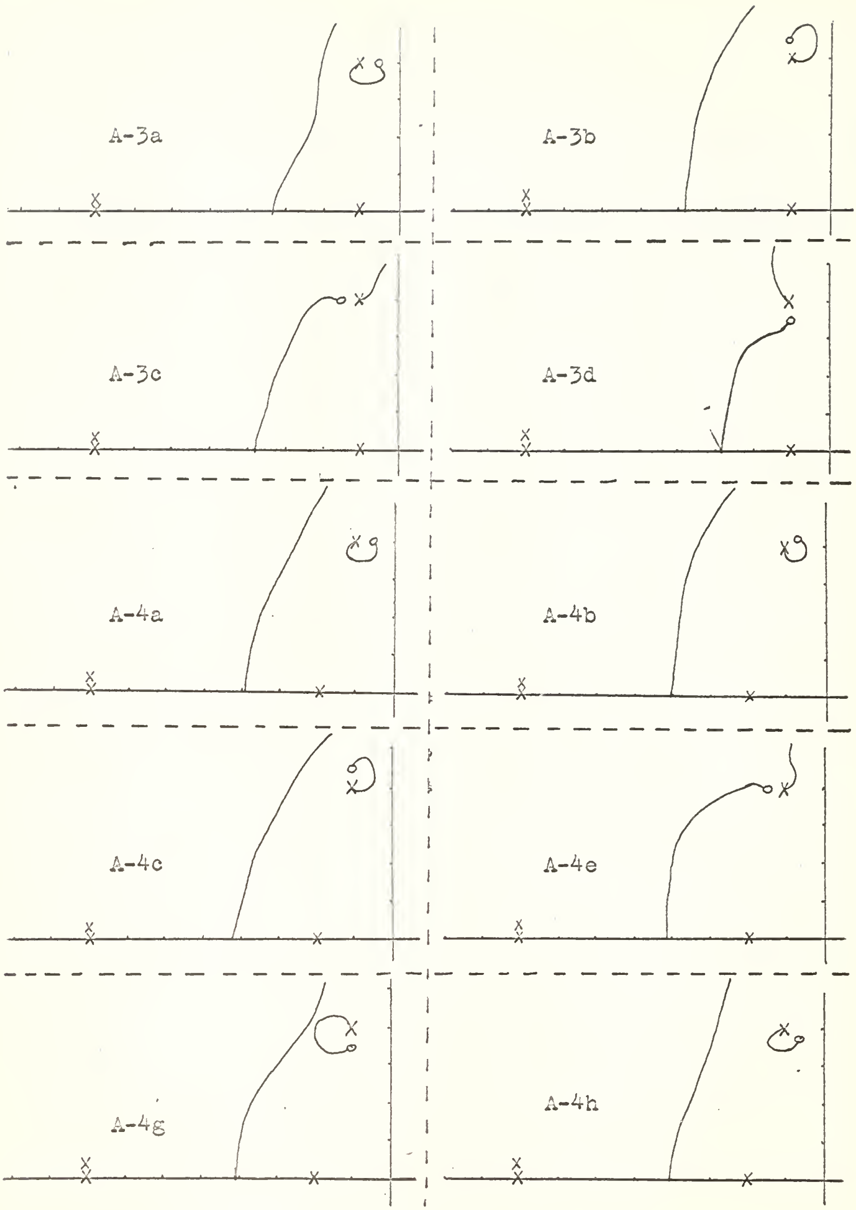


Fig. 10b Root Locus Data

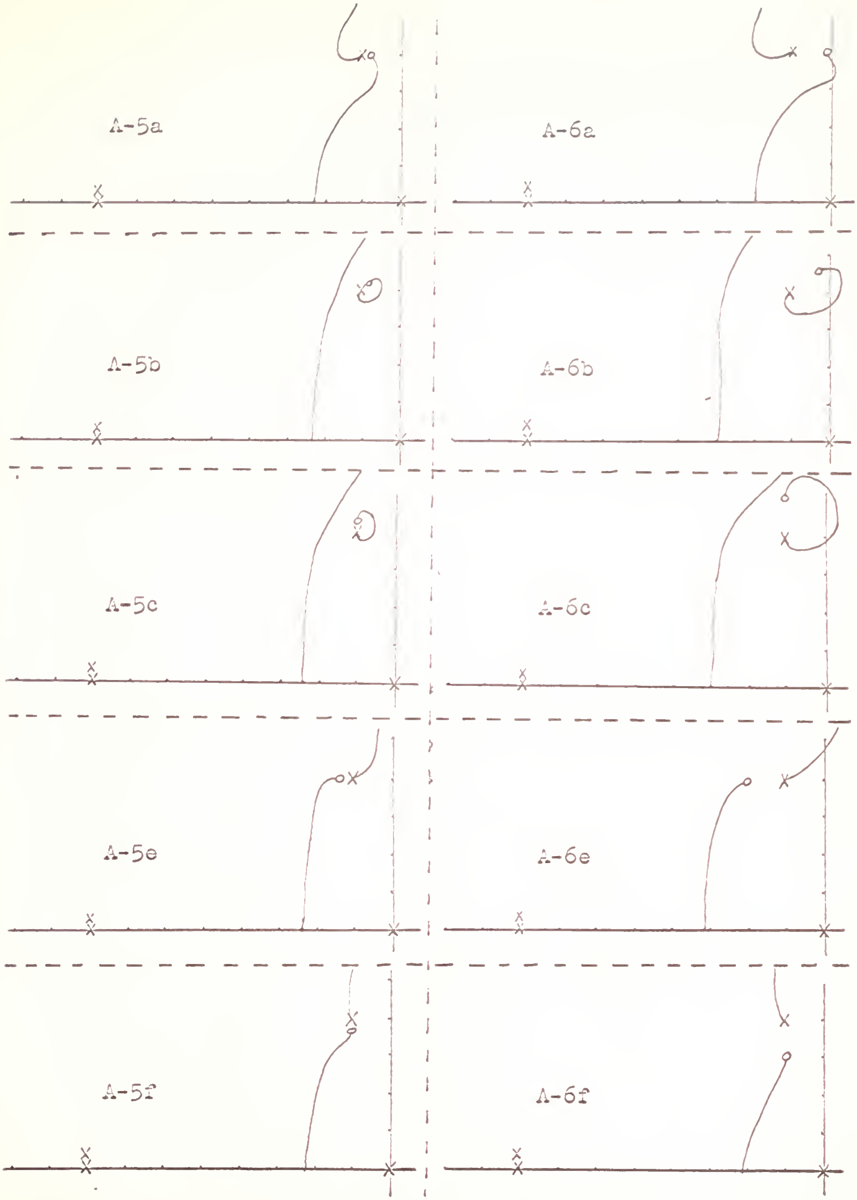


Fig. 10c Root Locus Data

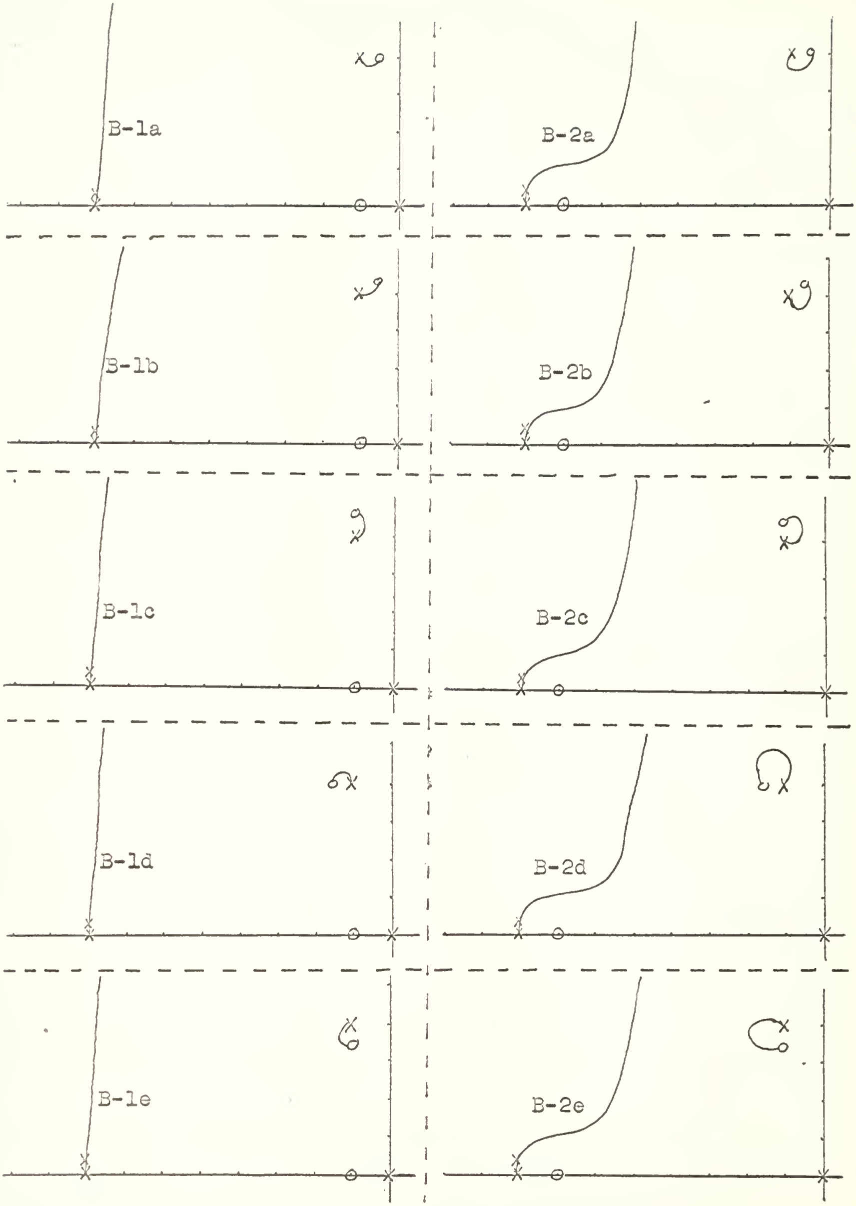


Fig. 10d Root Locus Data

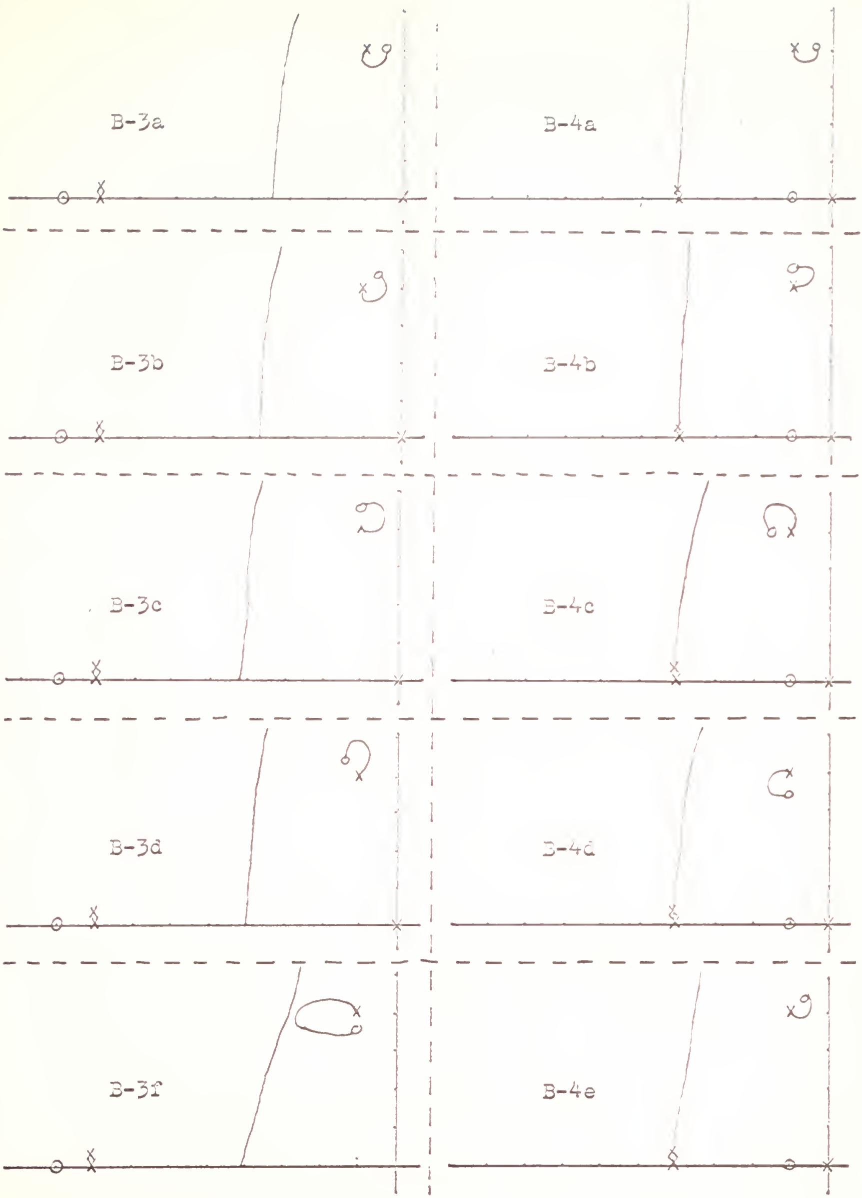


Fig. 10e Root Locus Data

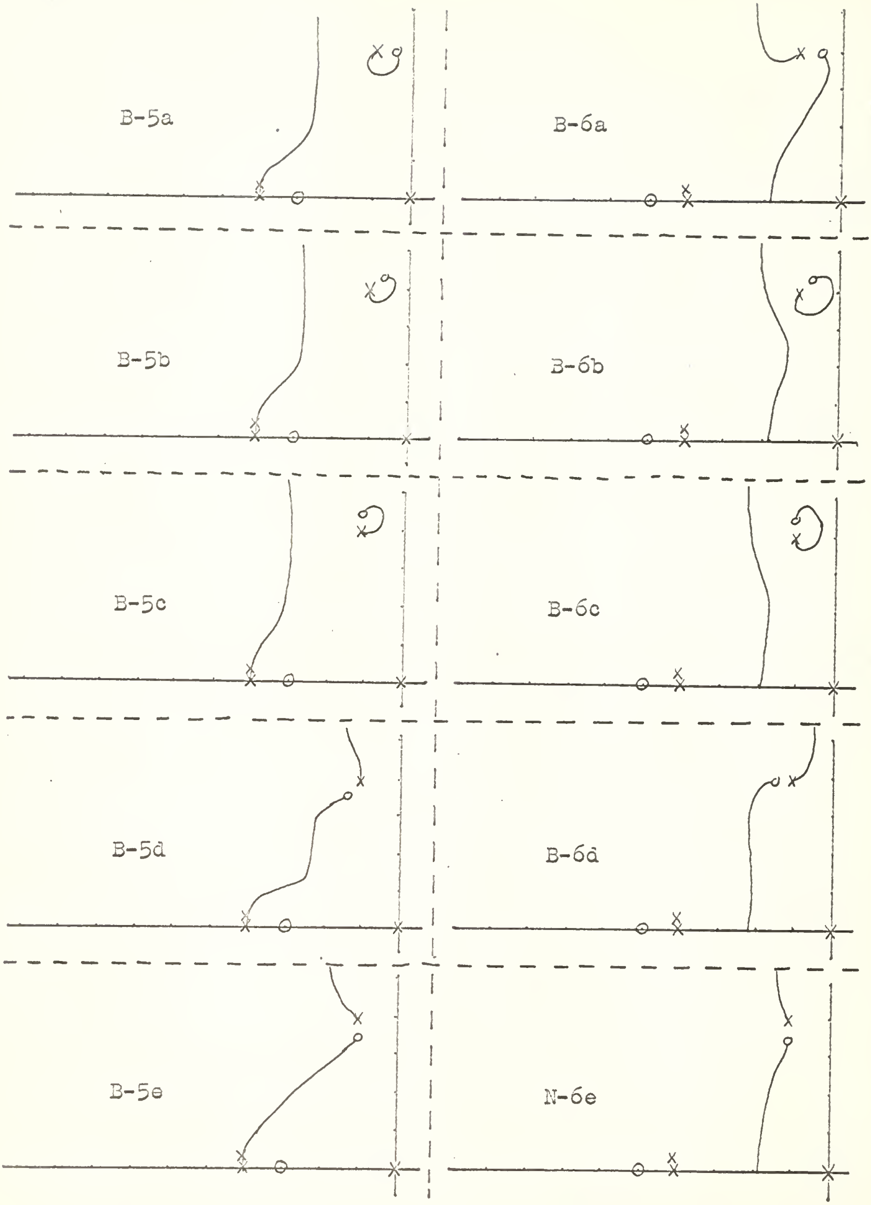


Fig. 10f Root Locus Data

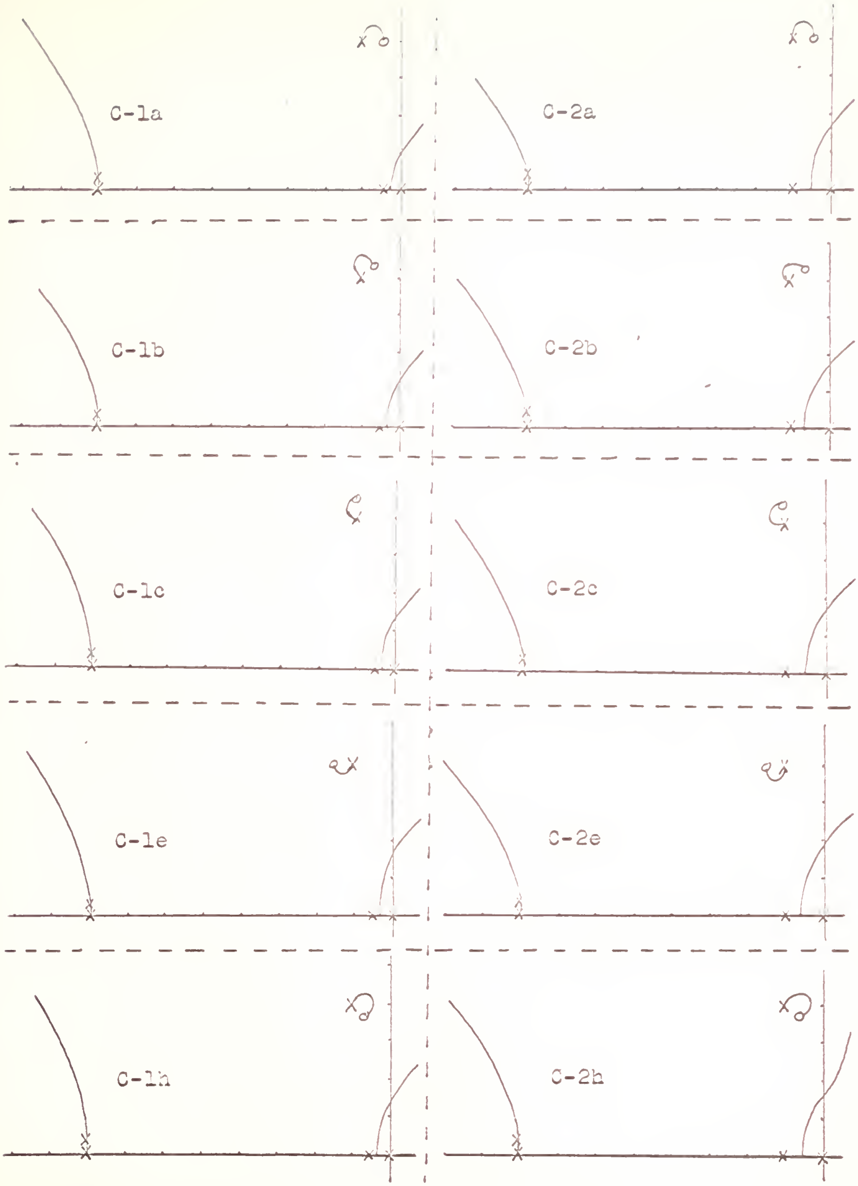


Fig. 10g Root Locus Data

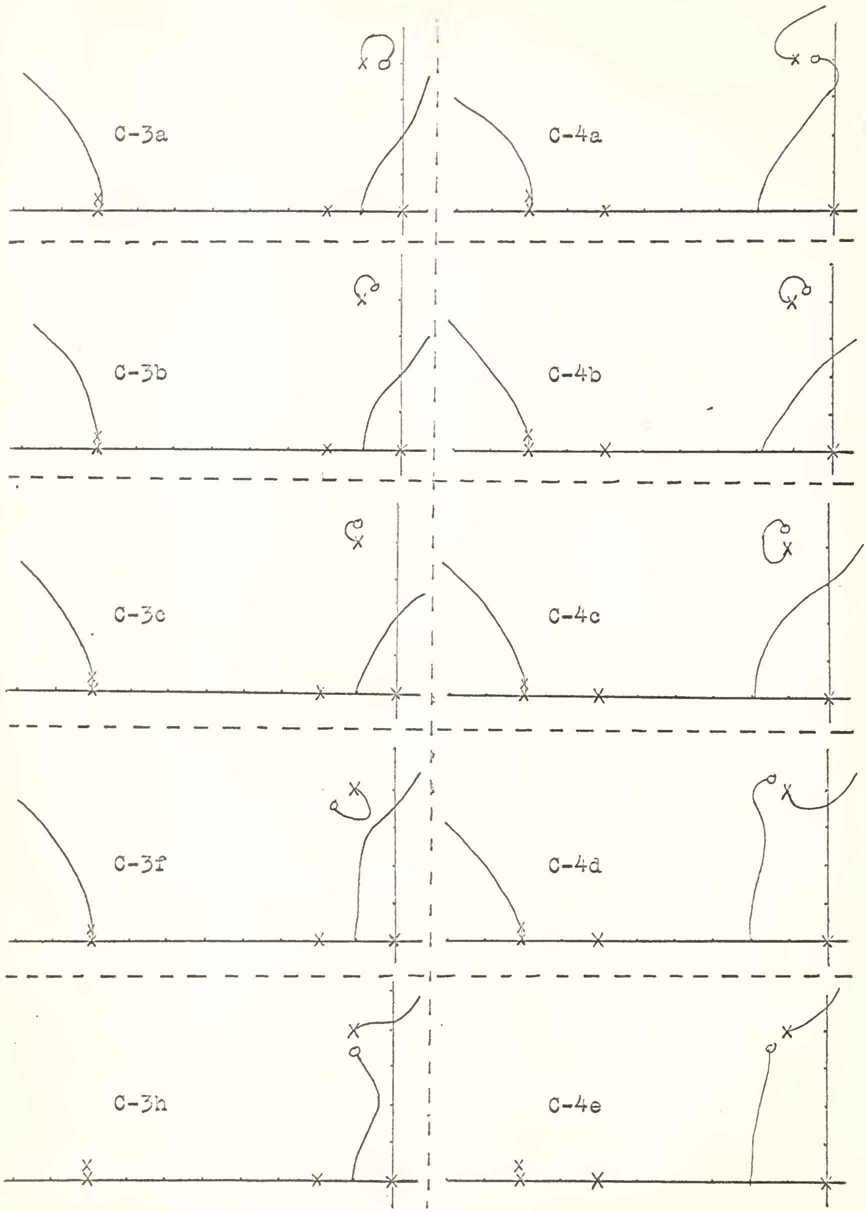


Fig. 10h Root Locus Data

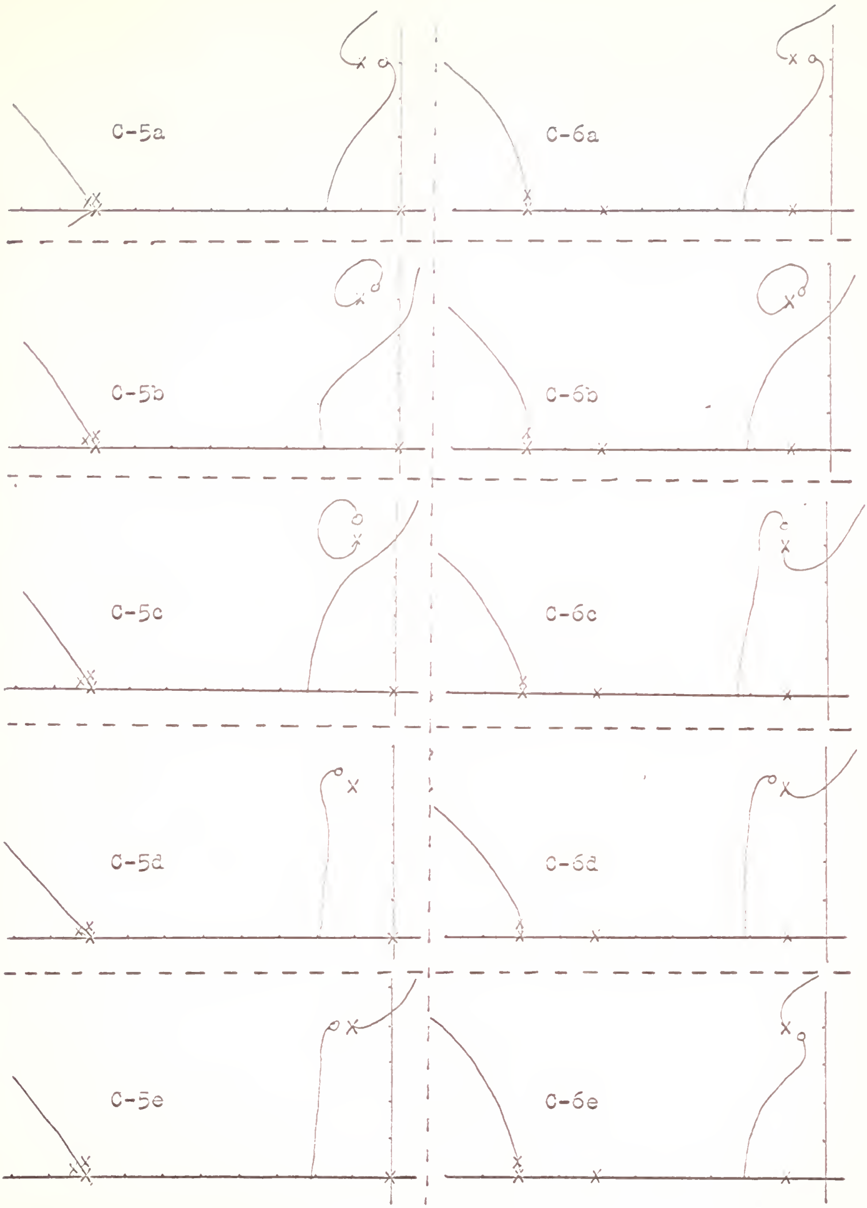


Fig. 10i Root Locus Data

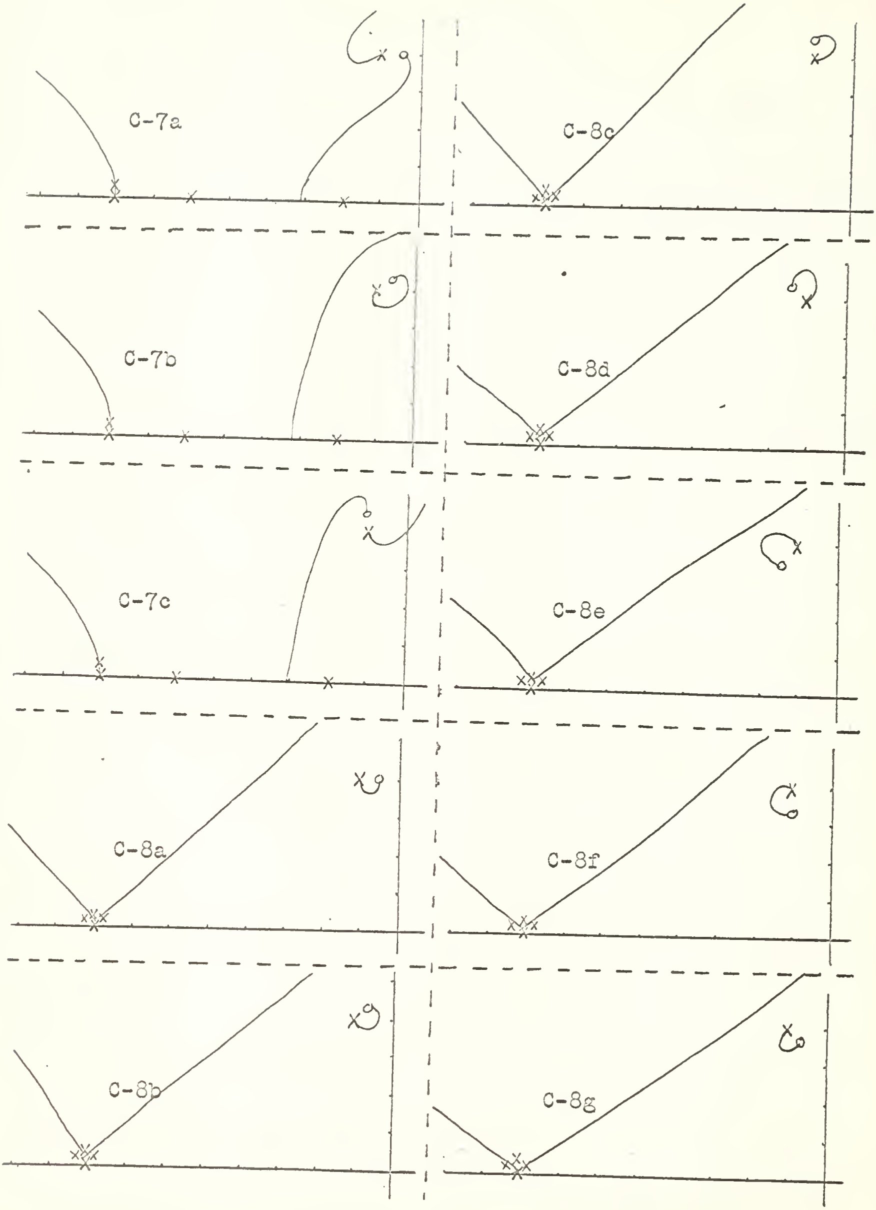


Fig. 10j Root Locus Data

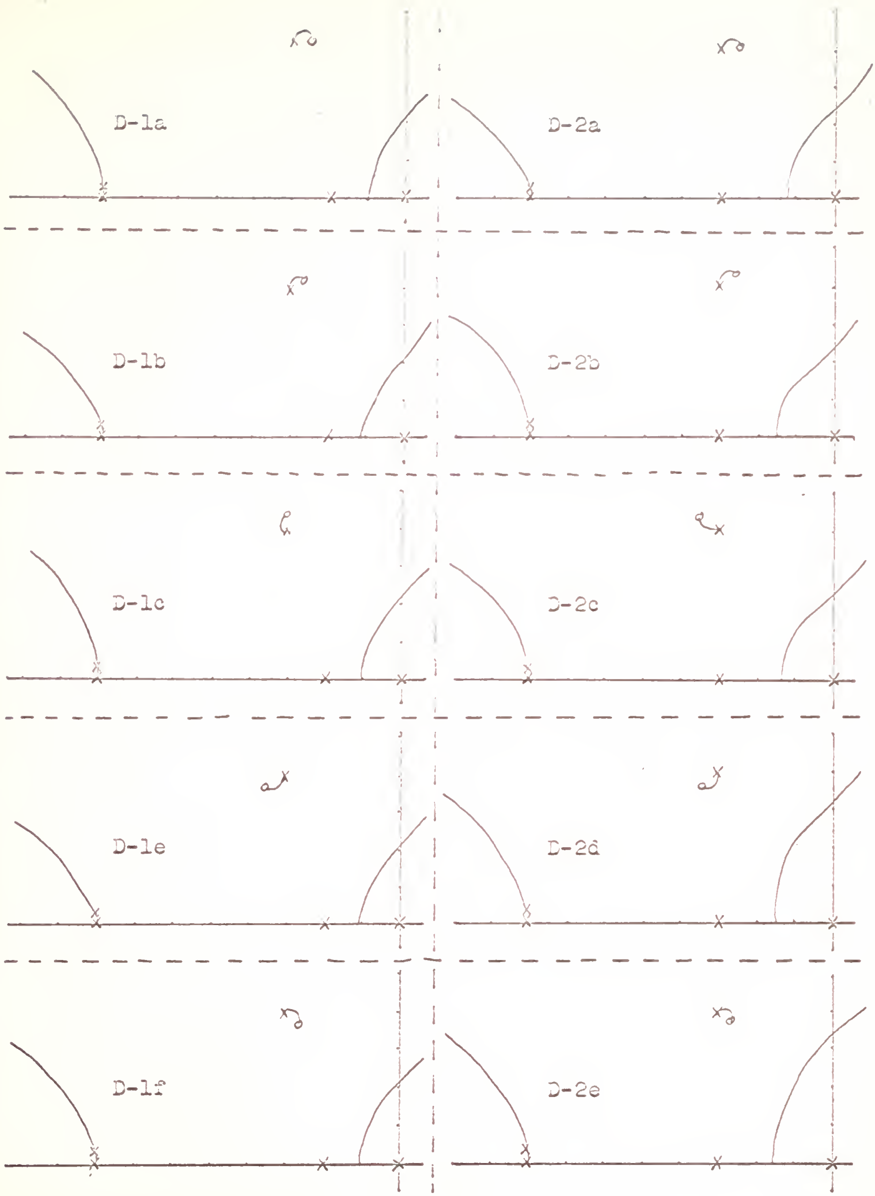


Fig. 10k Root Locus Data

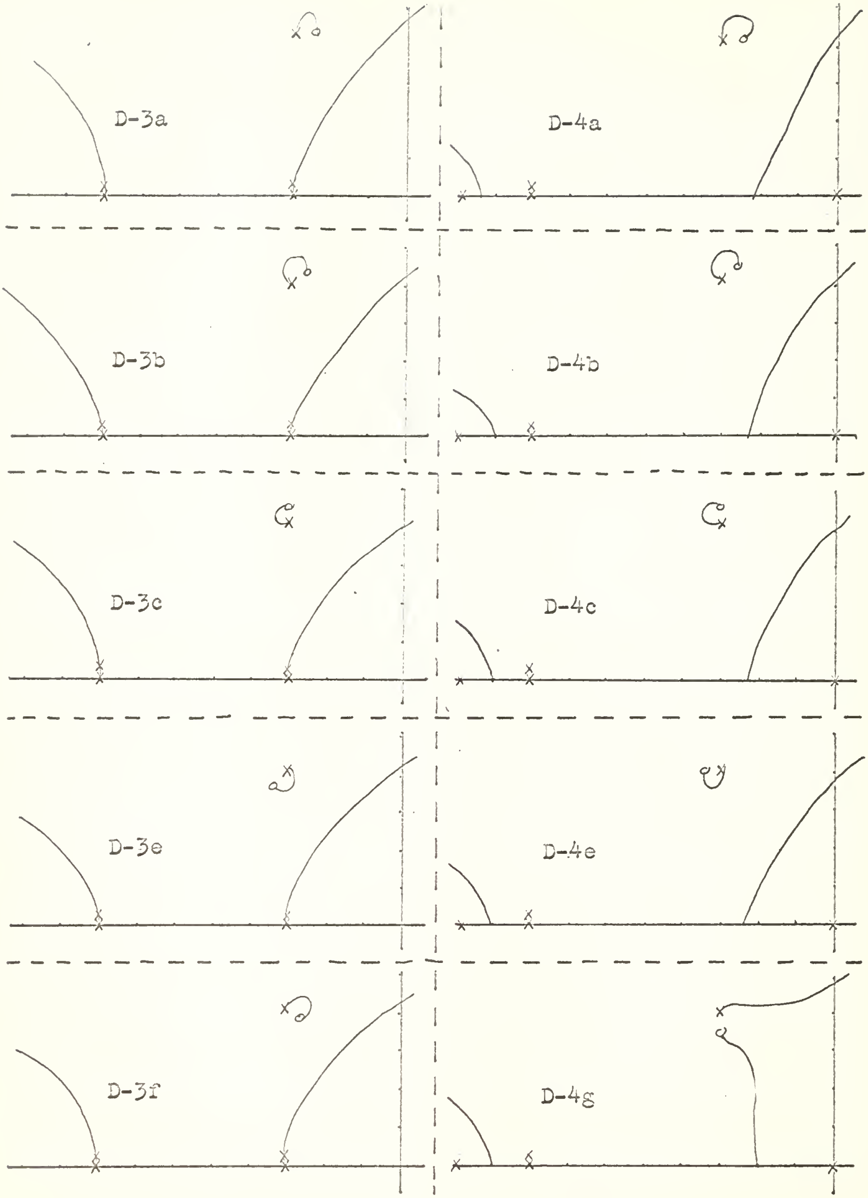


Fig. 101 Root Locus Data

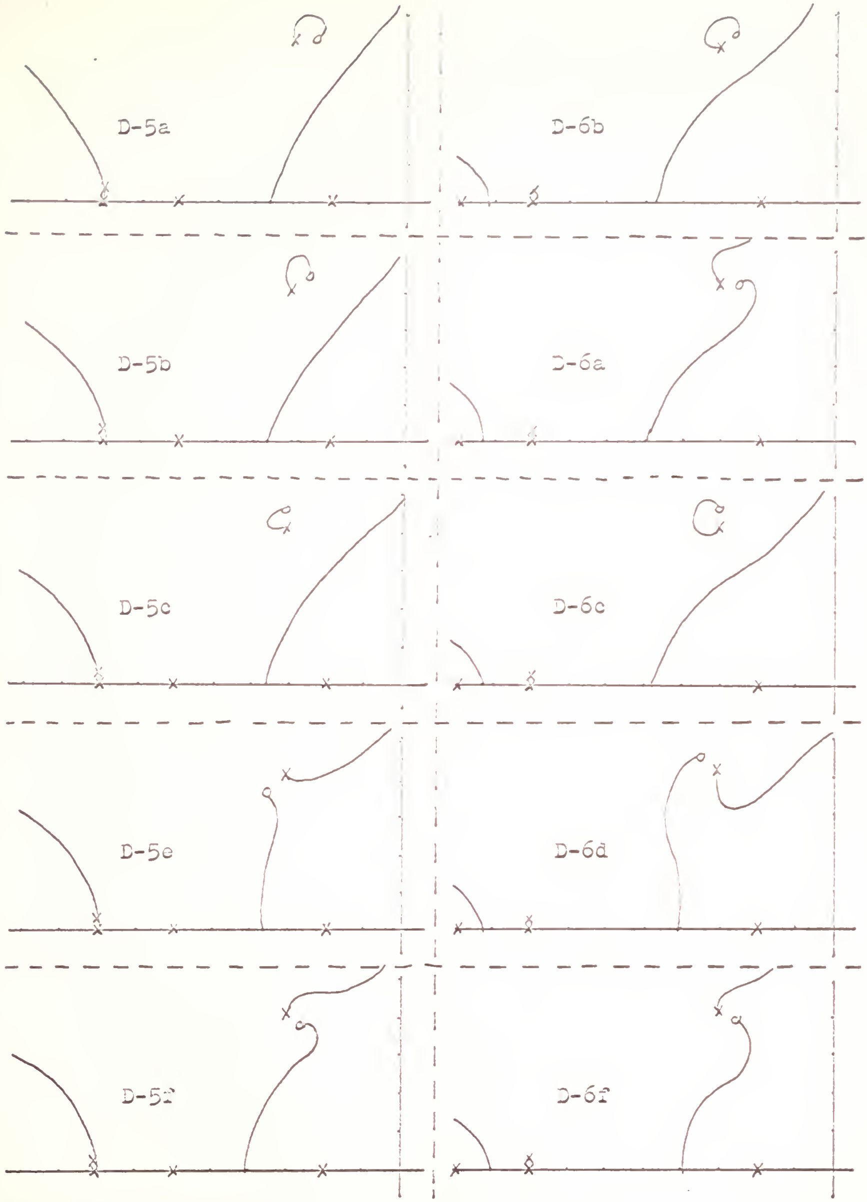


Fig. 10m Root Locus Data

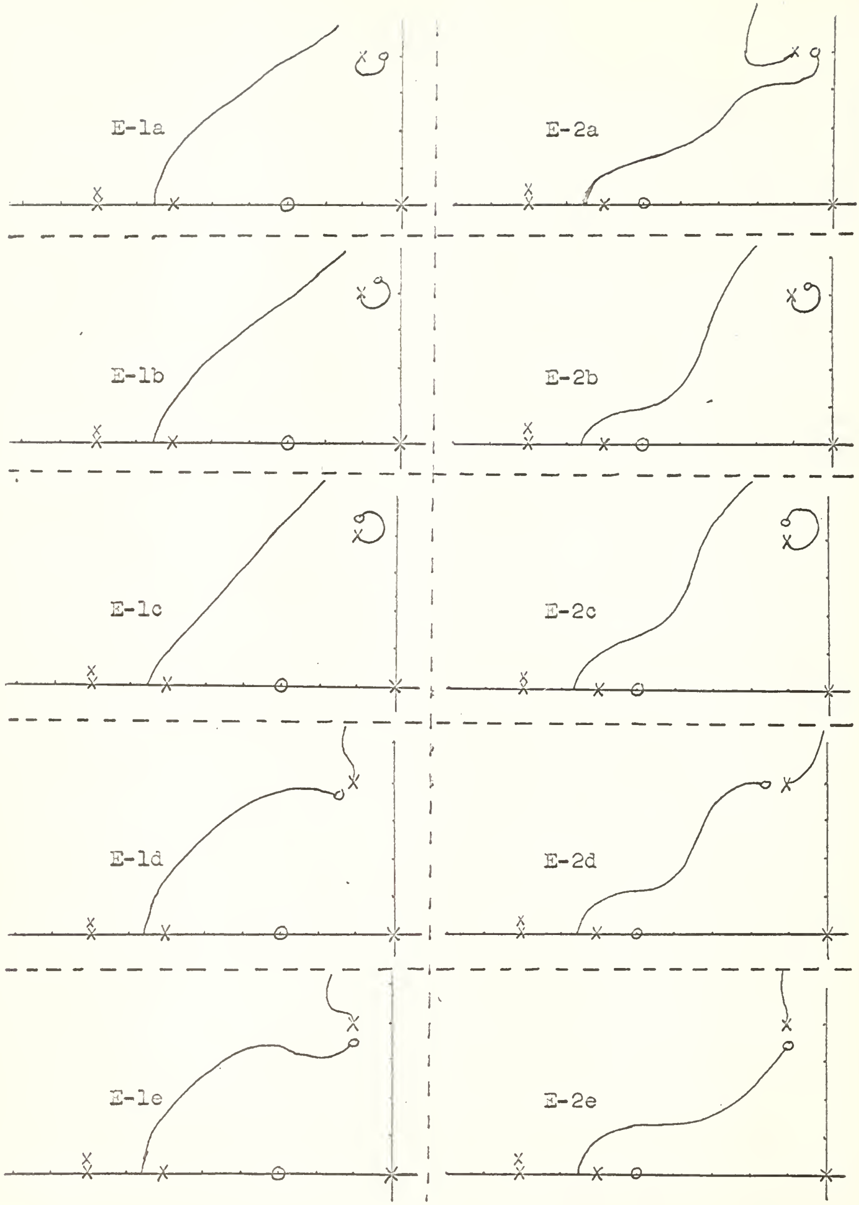


Fig. 10n Root Locus Data

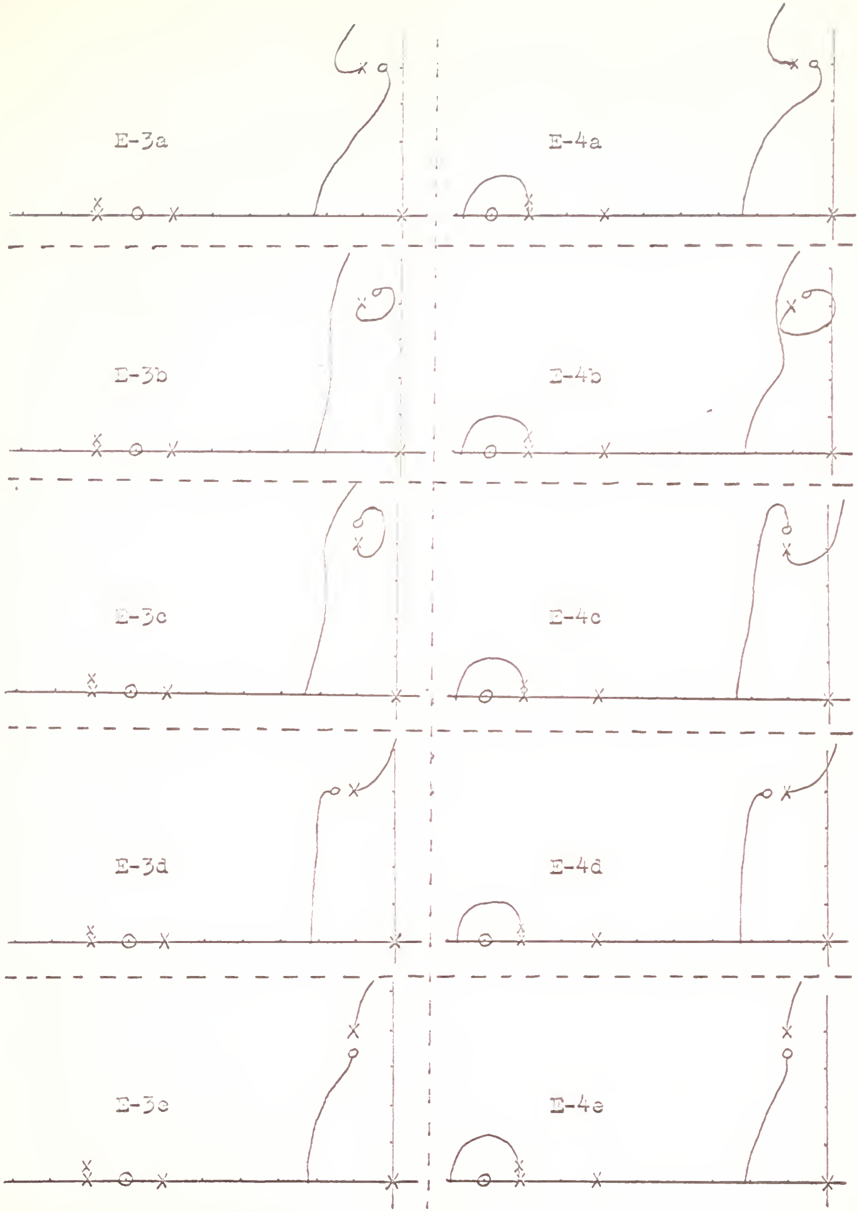


Fig. 10o Root Locus Data

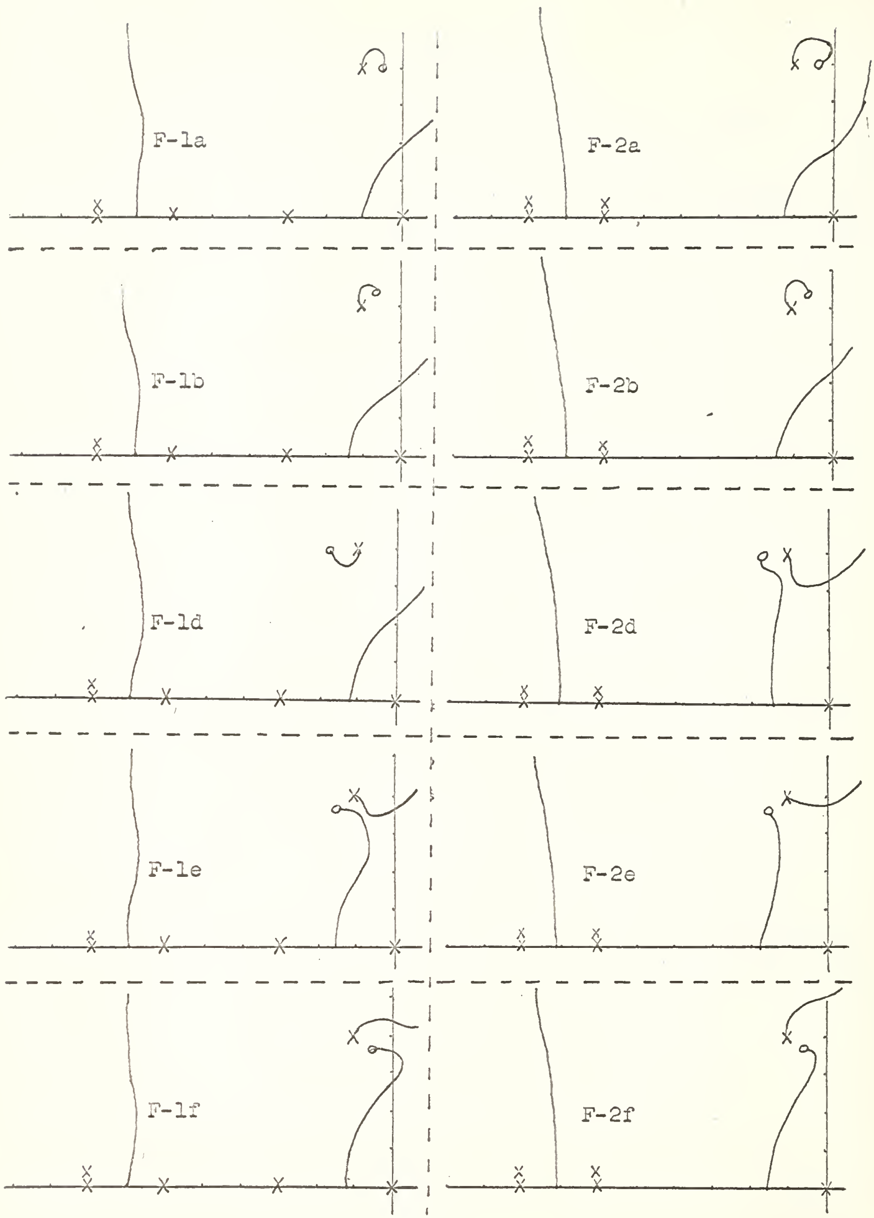


Fig. 10p Root Locus Data

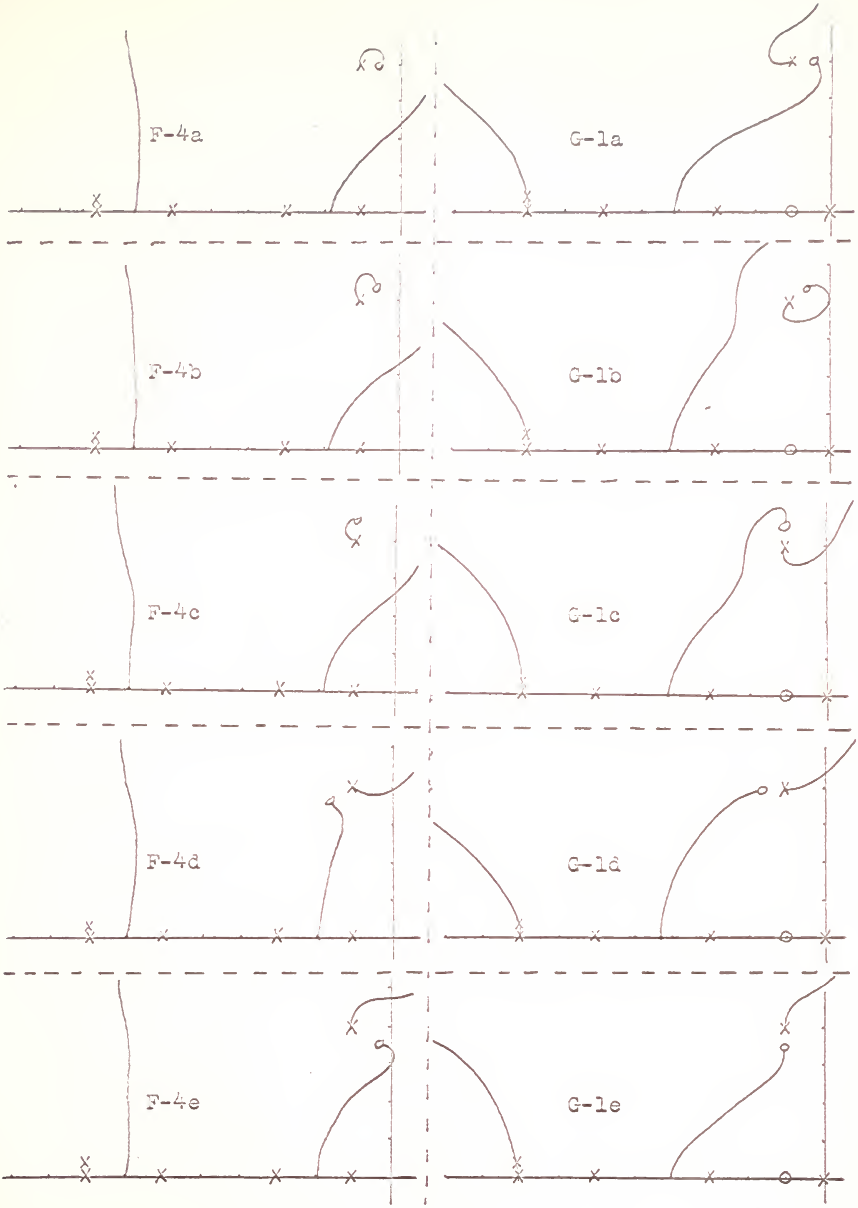


Fig. 10q Root Locus Data

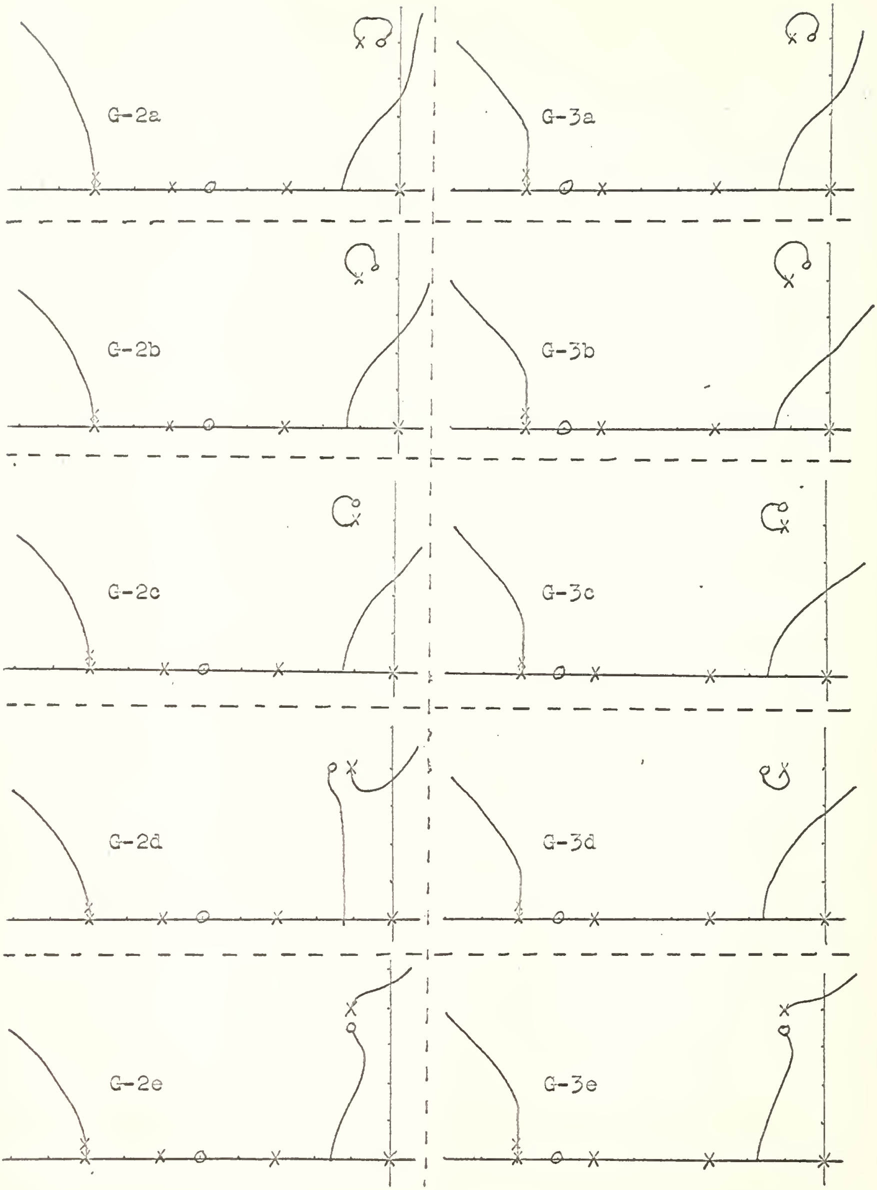


Fig. 10r Root Locus Data

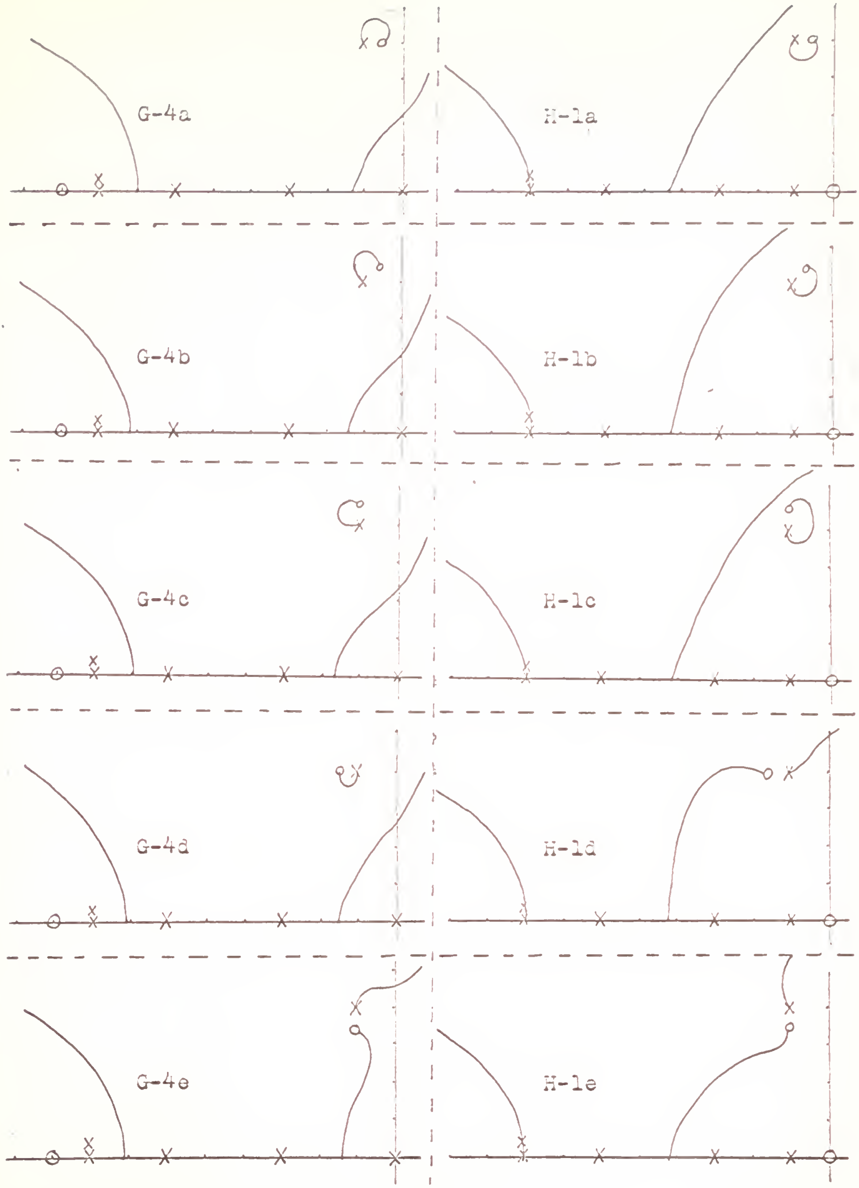


Fig. 10s Root Locus Data

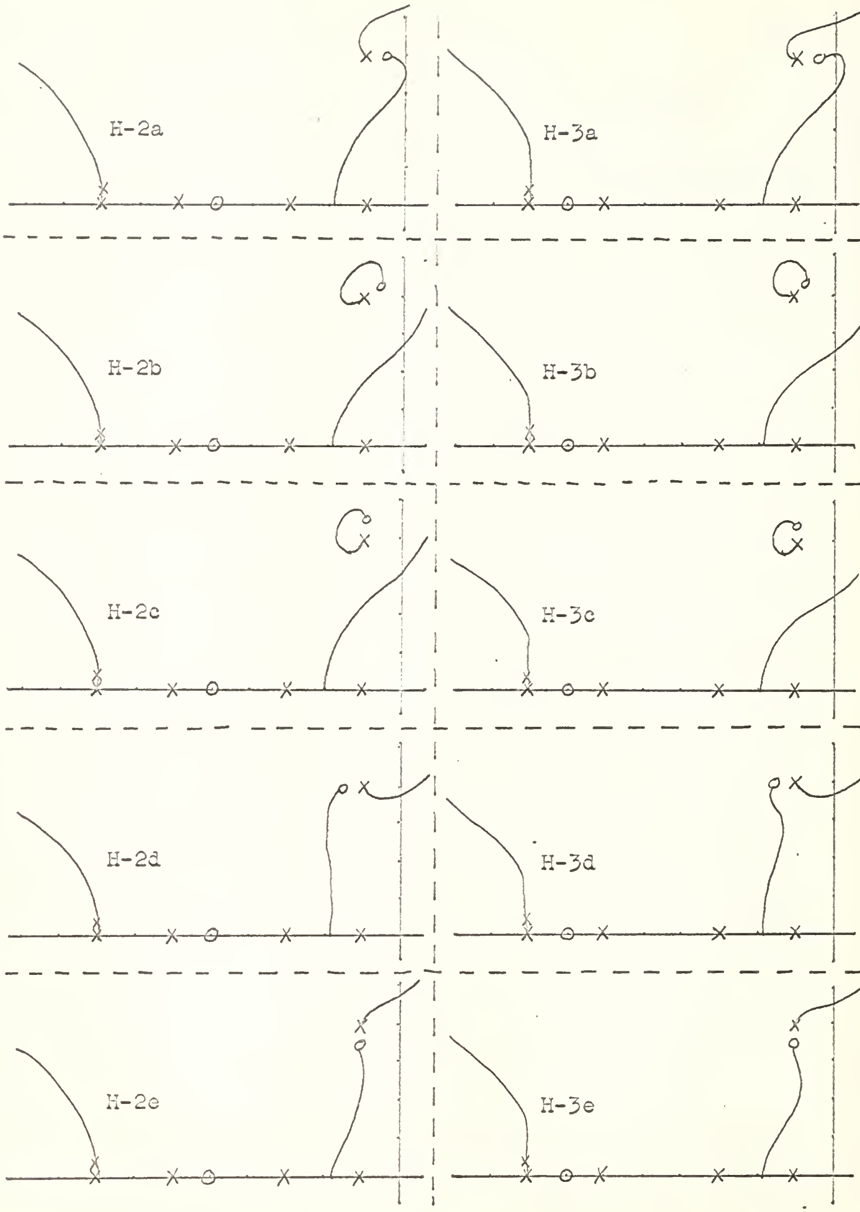


Fig. 10t Root Locus Data

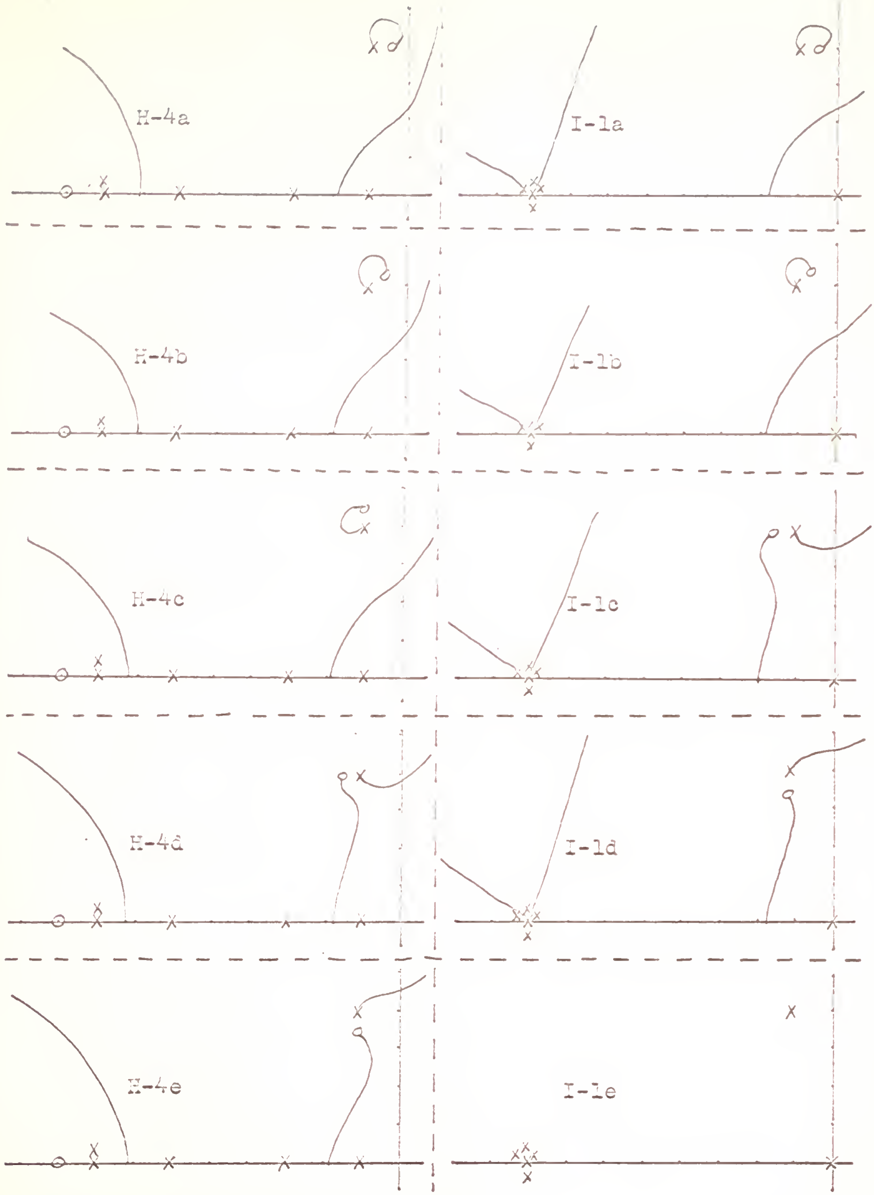


Fig. 10u Root Locus Data

2. Observations from Data

a. There is always a zero position angle θ in the first quadrant with respect to the complex pole which will cause the root locus from the complex pole to end on the near complex zero. This is observed in all cases.

b. If the root locus closes on the complex zero, then the locus from the pole moves in direction of least number of degrees (angle of emergence from pole compared with angle of emergence from zero).

c. The locus closes on complex zero if there is a root locus segment on negative real axis to the right (closest to the origin) of the real part of the complex conjugate pole. (C-1, C-2, D-1, D-2, D-3)

d. Just as the radial distance of zero had little effect on angle of emergence, it also has little effect on root locus plots except to make plots proportionately larger or smaller. (Compare Case A-1, A-5, and A-6).

e. The locus from complex pole closes on complex zero if asymptote vector is 90 degrees and the asymptote centroid is far enough out on negative real axis. (i.e., Number of pole minus number of zeros is equal to two). See cases B-1, B-2, B-3, and B-4.

TABLE 1

SUMMARY OF DATA*

Desig	θ	C,NC	Transfer Function; Other Data
A-1a	000	NC	$\dfrac{K(s+a-jb)(s+a+jb)}{s(s+8)^2(s+1-j4)(s+1+j4)}$
A-1b	045	C	
A-1c	090	C	$\angle K_p = 106°$
A-1d	135	NC	$\psi_c = 196°$, $\psi_z = 344°$
A-2a	000	NC	
A-2b	045	C	$\dfrac{K(s+a+jb)(s+a-jb)}{s(s+4)^2(s+1-j4)(s+1+j4)}$
A-2c	090	C	
A-2d	135	C	
A-2e	180	NC	
A-2f	225	NC	
A-2g	315	NC	$\angle K_p = 61°$ $\psi_c = 151$, $\psi_z = 29°$
A-3a	000	C	$\dfrac{K(s+a-jb)(s+a-jb)}{(s+1)(s+8)^2(s+1-j4)(s+1+j4)}$
A-3b	090	C	
A-3c	180	NC	
A-3d	270	NC	$\angle K_p = 120$ $\psi_c = 210$ $\psi_z = 330$
A-4a	000	C	$\dfrac{K(s+a-jb)(s+a+jb)}{(s+2)(s+8)^2(s+1-j4)(s+1+j4)}$
A-4b	045	C	
A-4c	090	C	
A-4d	135	C	
A-4e	180	NC	
A-4f	225	NC	
A-4g	270	C	$\angle K_p = 134$ $\psi_c = 224$ $\psi_z = 316$
A-4h	315	C	
A-5a	000	NC	Same as A-1 except complex zeros are on .25 circle.
A-5b	045	C	
A-5c	090	C	
A-5d	135	NC	
A-5e	180	NC	
A-5f	270	NC	
A-5g	315	NC	
A-6a	000	NC	Same as A-1 except complex zeros are on 1.0 circle.
A-6b	045	C	
A-6c	090	C	
A-6d	135	NC	
A-6e	180	NC	

*"Desig" is the designation of each root locus plot, θ (theta) is the angular position of the complex zero measured with respect to its adjacent complex pole, "C" indicates that the root locus from the complex pole closes or ends on its adjacent complex zero, "NC" indicates that the root locus does not close on the complex zero. Values of $\angle K_p$ were found by use of a spirule, and the values of ψ_c and ψ_z were calculated by use of the formulas developed in Chapter II with θ equal to 000 degrees.

Table 1 (Cont'd)

Desig	θ	C,NC	Transfer Function; Other Data
B-1a	000	C	$\frac{K(s+1)(s+a-jb)(s+a+jb)}{s(s+8)^2(s+1-j4)(s+1+j4)}$
B-1b	045	C	
B-1c	090	C	
B-1d	180	C	
B-1e	270	C	$\angle K_p = 196 \quad \psi_c = 286 \quad \psi_z = 254$
B-2a	000	C	$\frac{K(s+7)(s+a-jb)(s+a+jb)}{s(s+8)^2(s+1-j4)(s+1+j4)}$
B-2b	045	C	
B-2c	090	C	
B-2d	180	C	
B-2e	270	C	$\angle K_p = 170 \quad \psi_c = 260 \quad \psi_z = 280$
B-3a	000	C	$\frac{K(s+9)(s+a-jb)(s+a+jb)}{s(s+8)^2(s+1-j4)(s+1+j4)}$
B-3b	045	C	
B-3c	090	C	
B-3d	135	C	
B-3e	180	C	
B-3f	270	C	$\angle K_p = 132 \quad \psi_c = 222 \quad \psi_z = 318$
B-3g	315	C	
B-4a	000	C	$\frac{K(s+1)(s+a-jb)(s+a+jb)}{s(s+4)^2(s+1+j4)(s+1-j4)}$
B-4b	090	C	
B-4c	180	C	
B-4d	270	C	$\angle K_p = 150 \quad \psi_c = 240 \quad \psi_z = 300$
B-5a	000	C	$\frac{K(s+3)(s+a-jb)(s+a+jb)}{s(s+4)^2(s+1-j4)(s+1+j4)}$
B-5b	045	C	
B-5c	090	C	
B-5d	225	NC	
B-5e	270	NC	$\angle K_p = 124 \quad \psi_c = 214 \quad \psi_z = 326$
B-6a	000	NC	$\frac{K(s+5)(s+a+jb)(s+a-jb)}{s(s+4)^2(s+1-j4)(s+1+j4)}$
B-6b	045	C	
B-6c	090	C	
B-6d	180	NC	
B-6e	270	NC	
C-1a	000	C	
C-1b	045	C	$\frac{K(s+a+jb)(s+a-jb)}{s(s+.5)(s+8)^2(s+1-j4)(s+1+j4)}$
C-1c	090	C	
C-1d	135	C	
C-1e	180	C	
C-1f	225	C	
C-1g	270	C	$\angle K_p = 10 \quad \psi_c = 100 \quad \psi_z = 80$
C-1h	315	C	
C-2a	000	C	
C-2b	045	C	
C-2c	090	C	

Table 1 (Cont'd)

Desig	θ	C,NC	Transfer Function; Other Data
C-2d	135	C	$\frac{K(s+a+jb)(s+a-jb)}{s(s+1)(s+8)^2(s+1+j4)(s+1-j4)}$
C-2e	180	C	
C-2f	225	C	
C-2g	270	C	
C-2h	315	C	$\angle K_p = 18 \quad \psi_c = 108 \quad \psi_z = 72$
C-3a	000	C	$\frac{K(s+a+jb)(s+a-jb)}{s(s+2)(s+8)^2(s+1+j4)(s+1-j4)}$
C-3b	045	C	
C-3c	090	C	
C-3d	135	C	
C-3e	180	C	
C-3f	225	C	$\angle K_p = 30 \quad \psi_c = 120 \quad \psi_z = 60$
C-3g	270	NC	
C-3h	315	NC	
C-4a	000	NC	$\frac{K(s+a+jb)(s+a-jb)}{s(s+6)(s+8)^2(s+1+j4)(s+1-j4)}$
C-4b	045	C	
C-4c	090	C	
C-4d	135	NC	
C-4e	225	NC	$\angle K_p = 68 \quad \psi_c = 158 \quad \psi_z = 22$
C-4f	315	NC	
C-5a	000	NC	$\frac{K(s+a+jb)(s+a-jb)}{s(s+8)^3(s+1-j4)(s+1+j4)}$
C-5b	045	C	
C-5c	090	C	
C-5d	135	NC	
C-5e	180	NC	
C-5f	225	NC	$\angle K_p = 75 \quad \psi_c = 165 \quad \psi_z = 15$
C-5g	270	NC	
C-5h	315	NC	
C-6a	000	NC	$\frac{K(s+a+jb)(s+a-jb)}{(s+1)(s+6)(s+8)^2(s+1-j4)(s+1+j4)}$
C-6b	045	C	
C-6c	090	NC	
C-6d	135	NC	
C-6e	225	NC	$\angle K_p = 82 \quad \psi_c = 172 \quad \psi_z = 8$
C-7a	000	NC	$\frac{K(s+a+jb)(s+a-jb)}{(s+2)(s+6)(s+8)^2(s+1+j4)(s+1-j4)}$
C-7b	045	C	
C-7c	090	NC	$\angle K_p = 95 \quad \psi_c = 185 \quad \psi_z = 355$
C-8a	000	C	$\frac{K(s+a-jb)(s+a+jb)}{(s+8)^4(s+1+j4)(s+1-j4)}$
C-8b	045	C	
C-8c	090	C	
C-8d	135	C	
C-8e	225	C	
C-8f	270	C	$\angle K_p = 145 \quad \psi_c = 239 \quad \psi_z = 301$
C-8g	315	C	

Table 1 (Cont'd)

Desig	θ	C,NC	Transfer Function; Other Data
D-1a	000	C	$\frac{K(s+a-jb)(s+a+jb)}{s(s+2)(s+8)^2(s+3+j4)(s+3-j4)}$
D-1b	045	C	
D-1c	090	C	
D-1d	135	C	
D-1e	225	C	$\angle K_p = 321 \quad \psi_c = 51 \quad \psi_z = 129$
D-1f	315	C	
D-2a	000	C	$\frac{K(s+a-jb)(s+a+jb)}{s(s+3)(s+8)^2(s+3-j4)(s+3+j4)}$
D-2b	045	C	
D-2c	135	C	
D-2d	225	C	$\angle K_p = 333 \quad \psi_c = 63 \quad \psi_z = 117$
D-2e	315	C	
D-3a	000	C	$\frac{K(s+a-jb)(s+a+jb)}{(s+3)^2(s+8)^2(s+3+j4)(s+3-j4)}$
D-3b	045	C	
D-3c	090	C	
D-3d	135	C	
D-3e	225	C	$\angle K_p = 12 \quad \psi_c = 102 \quad \psi_z = 78$
D-3f	315	C	
D-4a	000	C	$\frac{K(s+a+jb)(s+a-jb)}{s(s+8)^2(s+10)(s+3+j4)(s+3-j4)}$
D-4b	045	C	
D-4c	090	C	
D-4d	135	C	
D-4e	180	C	
D-4f	225	C	$\angle K_p = 36 \quad \psi_c = 126 \quad \psi_z = 54$
D-4g	270	NC	
D-4h	315	C	
D-5a	000	C	$\frac{K(s+a-jb)(s+a+jb)}{(s+2)(s+6)(s+8)^2(s+3-j4)(s+3+j4)}$
D-5b	045	C	
D-5c	090	C	
D-5d	135	C	
D-5e	225	NC	$\angle K_p = 36 \quad \psi_c = 126 \quad \psi_z = 54$
D-5f	315	NC	
D-6a	000	NC	$\frac{K(s+a+jb)(s+a-jb)}{(s+2)(s+8)^2(s+10)(s+3+j4)(s+3-j4)}$
D-6b	045	C	
D-6c	090	C	
D-6d	135	NC	
D-6e	225	NC	$\angle K_p = 58 \quad \psi_c = 148 \quad \psi_z = 32$
D-6f	315	NC	
E-1a	000	C	$\frac{K(s+3)(s+a-jb)(s+a+jb)}{s(s+6)(s+8)^2(s+1-j4)(s+1+j4)}$
E-1b	045	C	
E-1c	090	C	
E-1d	225	NC	$\angle K_p = 132 \quad \psi_c = 222 \quad \psi_z = 318$
E-1e	270	NC	

Table 1 (Cont'd)

Desig	θ	C,NC	Transfer Function; Other Data
E-2a	000	C	$\frac{K(s+5)(s+a-jb)(s+a+jb)}{s(s+6)(s+8)^2(s+1-j4)(s+1+j4)}$
E-2b	045	C	
E-2c	090	C	
E-2d	180	NC	
E-2e	270	NC	$\angle K_p = 113$ $\psi_c = 203$ $\psi_z = 337$
E-3a	000	NC	$\frac{K(s+7)(s+a-jb)(s+a+jb)}{s(s+6)(s+8)^2(s+1-j4)(s+1+j4)}$
E-3b	045	C	
E-3c	090	NC	
E-3d	180	NC	
E-3e	270	NC	$\angle K_p = 101$ $\psi_c = 191$ $\psi_z = 349$
E-4a	000	NC	$\frac{K(s+9)(s+a-jb)(s+a+jb)}{s(s+6)(s+8)^2(s+1-j4)(s+1+j4)}$
E-4b	045	C	
E-4c	090	NC	
E-4d	180	NC	
E-4e	270	NC	$\angle K_p = 95$ $\psi_c = 185$ $\psi_z = 355$
F-1a	000	C	$\frac{K(s+a-jb)(s+a+jb)}{s(s+3)(s+6)(s+8)^2(s+1-j4)(s+1+j4)}$
F-1b	045	C	
F-1c	090	C	
F-1d	180	C	
F-1e	225	NC	$\angle K_p = 1$ $\psi_c = 91$ $\psi_z = 89$
F-1f	315	NC	
F-2a	000	NC	$\frac{K(s+a-jb)(s+a+jb)}{s(s+6)^2(s+8)^2(s+1-j4)(s+1+j4)}$
F-2b	045	C	
F-2c	090	C	
F-2d	180	NC	
F-2e	225	NC	$\angle K_p = 27$ $\psi_c = 117$ $\psi_z = 63$
F-2f	315	NC	
F-3a	000	NC	$\frac{K(s+a+jb)(s+a-jb)}{s(s+8)^2(s+10)^2(s+1-j4)(s+1+j4)}$
F-3b	045	C	
F-3c	090	C	
F-3d	180	NC	
F-3e	315	NC	$\angle K_p = 58$ $\psi_c = 148$ $\psi_z = 32$
F-4a	000	C	$\frac{K(s+a-jb)(s+a+jb)}{(s+1)(s+3)(s+6)(s+8)^2(s+1-j4)(s+1+j4)}$
F-4b	045	C	
F-4c	090	C	
F-4d	225	NC	
F-4e	315	NC	$\angle K_p = 20$ $\psi_c = 110$ $\psi_z = 70$
G-1a	000	NC	$\frac{K(s+1)(s+a-jb)(s+a-jb)}{s(s+3)(s+6)(s+8)^2(s+1-j4)(s+1+j4)}$
G-1b	045	C	
G-1c	090	NC	
G-1d	180	NC	
G-1e	270	NC	$\angle K_p = 95$ $\psi_c = 185$ $\psi_z = 355$

Table 1 (Cont'd)

Desig	θ	C,NC	Transfer Function; Other Data
G-2a	000	C	$\dfrac{K(s+5)(s+a-jb)(s+a+jb)}{s(s+3)(s+6)(s+8)^2(s+1-j4)(s+1+j4)}$ $\angle K_p = 50$ $\psi_c = 140$ $\psi_z = 40$
G-2b	045	C	
G-2c	090	C	
G-2d	180	NC	
G-2e	270	NC	
G-3a	000	C	$\dfrac{K(s+a-jb)(s+a+jb)(s+7)}{s(s+3)(s+6)(s+8)^2(s+1-j4)(s+1+j4)}$ $\angle K_p = 36$ $\psi_c = 126$ $\psi_z = 54$
G-3b	045	C	
G-3c	090	C	
G-3d	180	C	
G-3e	270	NC	
G-4a	000	C	$\dfrac{K(s+9)(s+a-jb)(s+a+jb)}{s(s+3)(s+6)(s+8)^2(s+1-j4)(s+1+j4)}$ $\angle K_p = 30$ $\psi_c = 120$ $\psi_z = 60$
G-4b	045	C	
G-4c	090	C	
G-4d	180	C	
G-4e	270	NC	
H-1a	000	C	$\dfrac{K s(s+a-jb)(s+a+jb)}{(s+1)(s+3)(s+6)(s+8)^2(s+1-j4)(s+1+j4)}$ $\angle K_p = 123$ $\psi_c = 213$ $\psi_z = 327$
H-1b	045	C	
H-1c	090	C	
H-1d	180	NC	
H-1e	270	NC	
H-2a	000	NC	$\dfrac{K(s+5)(s+a-jb)(s+a+jb)}{(s+1)(s+3)(s+6)(s+8)^2(s+1-j4)(s+1+j4)}$ $\angle K_p = 60$ $\psi_c = 150$ $\psi_z = 30$
H-2b	045	C	
H-2c	090	C	
H-2d	180	NC	
H-2e	270	NC	
H-3a	000	NC	$\dfrac{K(s+7)(s+a-jb)(s+a+jb)}{(s+1)(s+3)(s+6)(s+8)^2(s+1-j4)(s+1+j4)}$ $\angle K_p = 52$ $\psi_c = 142$ $\psi_z = 38$
H-3b	045	C	
H-3c	090	C	
H-3d	180	NC	
H-3e	270	NC	
H-4a	000	NC	$\dfrac{K(s+9)(s+a-jb)(s+a+jb)}{(s+1)(s+3)(s+6)(s+8)^2(s+1-j4)(s+1+j4)}$
H-4b	045	C	
H-4c	090	C	
H-4d	180	NC	
H-4e	270	NC	
I-1a	000	C	$\dfrac{K(s+a-jb)(s+a+jb)}{s(s+10)^5(s+1+j4)(s+1-j4)}$ $\angle K_p = 15$ $\psi_c = 105$ $\psi_z = 75$
I-1b	045	C	
I-1c	180	NC	
I-1d	270	NC	

CHAPTER IV

CONCLUSIONS

Two analytical methods for obtaining the angle of emergence from a complex pole or zero have been presented. In the first of these methods, the origin was transferred to the complex pole or zero (w-plane) and then the angle of emergence was defined by equations (2-2a) and (2-3a).

The second method was to determine the residue angle by means of Heaviside's expansion theorem and then simply add 180 degrees to the residue angle to obtain the angle of emergence.

The following equations are exact, i.e., no approximations were used.

$$\psi_c = \angle K_{pc} + 180 \qquad (2\text{-}5)$$

$$\psi_z = \angle K_z + 180 \qquad (2\text{-}7a)$$

The easiest method to obtain the angle of emergence is to find the residue angle with a spirule, then add 180 degrees.

Other relations developed in Chapter 2 are:

$$\psi_c = \theta + \psi_p - 90 \qquad (2\text{-}13)$$

$$\psi_z = -\psi_c + 2\theta + 180 \qquad (2\text{-}19)$$

$$\psi_z = \theta - \angle K_p + 90 \qquad (2\text{-}21)$$

$$\psi_z - \psi_c = -2\angle K_p \qquad (2\text{-}22)$$

and since $\psi_p = \angle K_p - 180$

$$\psi_c = \theta + \angle K_p + 90 \qquad (3\text{-}1)$$

It must be remembered that these equations are only approximate with the linearizing assumptions and approximations which have been made. Figures 7 and 8 are computer solutions of the angle of emergence vs. radial position of complex zero and the angle of emergence vs. zero positionangle respectively, and thus verifies the linear approximations for the assumed separation distance between complex conjugate poles.

Thus knowing the angle of emergence from a complex pole ψ_c, the position angle θ of the neighboring complex zero, the angle of emergence from the complex zero is readily obtained by equation (2-19). Similarly if the angle of emergence from a complex pole of an uncompensated system is known, the effect of adding complex zeros on the angle of emergence is obtained by using equation (2-13). Equation (2-22) reveals that the difference between the angles of emergence from complex pole and zero is proportional to the "uncompensated" residue angle and is independent of zero position angle θ.

The angle of emergence from complex pole and zero help to predict the manner in which the loci leave the pole or zero. A vector is drawn from the pole in direction of the angle of emergence and a second vector is drawn from the zero in the direction of its angle of emergence. Then the tails of these vectors are extended until they intersect at a point and thus defines an angle less than or equal to 180 degrees. It was observed that whenever the root locus from a pole closes on its adjacent zero, the loci moves in such a direction so as to be contained within the angle defined by the angle of emergence vectors.

It appears that there is always a zero position angle between 0 and 90 degrees (the first quadrant with respect to the complex pole) such that the root locus from the complex pole closes on the complex zero. Thus a self-adaptive zero compensator could be located within this 90 degree sector and effectively "compensate" the poles. The residues from the roots of these closed root loci would be negligible compared with the dominant roots of the control system.

BIBLIOGRAPHY

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APPENDIX I

Two digital computer programs were used. The first was "Program ANGMERG" to determine the angle of emergence for various pole and zero locations. Figures 7 and 8 were outputs of this program which verified analytical developments of Chapter II.

The second digital computer program was "Program Eqn" which is a program designed to plot root locus for any given characteristic equation (up to order 30). This program was originally programmed by LT Joseph D. Fennick, Jr., USN. A few changes in the original subroutine were made to facilitate data input, reduce printout, and speed up graph plotting time. Figures 11, 12, and 13 are sample outputs of this program. Table 2 contains all of the input data (characteristic equations) for each of the 215 individual cases studied.

```
..JOB0414F,REES, E.G.
      PROGRAM ANGMERG
C     THIS PROGRAM CALCULATES THE ANGLE OF EMERGENCE FOR VARIOUS
C     POLE AND ZERO LOCATIONS.  MULTIPLE RUNS CAN BE MADE BY
C     INDICATING NUMBER OF RUNS AND ADDING ADDITIONAL DATA CARDS
C     REAL AND IMAGINERY PARTS OF POLES AND ZEROS ARE READ IN
C     WITH STIPULATION THAT REP(1) AND YMP(1) BE COMPLEX POLE ABOUT
C     WHICH THE ANGLE OF EMERGENCE IS CALCULATED.  COMPENSATION
C     ZEROS ARE NOT READ IN, BUT ARE GENERATED IN THE PROGRAM.
C     LET REP(2) AND YMP(2) BE CONJUGATE OF REP(1) AND YMP(1)
C     PART OF THE PRINT OUT AND GRAPHS CAN BE SUPPRESSED BY
C     CHANGING VALUES OF W AND V RESPECTIVELY.  FOR A CURVE OF
C     RADIUS VS ANGLE OF EMERGENCE AT A PARTICULAR ANGLE OF
C     COMPENSATION ZERO FROM COMPLEX POLE, ANGLE AAG IS READ IN
C     IN RADIANS.  TWO GRAPHS OF ANGLE OF EMERGENCE VS ANGLE
C     POSITION OF COMPENSATING ZEROS AT A GIVEN RADIUS IS AVAILABLE.
C     RR1 AND RR2 ARE THE RADII FOR THESE CURVES. ADDITIONAL
C     SYMBOLS ARE-- CE TROID, THE INTERSECTION OF ASYMPTOTES,,
C     THETA -- ANGLE OF ASYMPTOTES FROM HORIZONTAL WHEN COM PENSATION
C     ZEROS ARE ADDED,, ZETA - ANGLE OF ASYMPTOTES FROM HORIZONTAL
C     WITHOUT COMPENSATING ZEROS,, ANGZ - SUMMATION OF ANGLES TO
C      ALL ZEROS FROM COMPLEX POLE,, ANGP - SUMMATION OF ANGLES TO
C     ALL POLES FROM COMPLEX POLE.
C     DATA READ IN AS FOLLOWS......
C         NOS    I2    NUMBER OF RUNS
C         M,N    2I2     NO. OF POLES M, NO. OF NON. COMP. ZEROS N
C         REP(I)   8F10.3    REAL PART OF POLES
C        YMP(I)   8F10.3     IMAG. PART OF POLES
C        REZ(I)    8F10.3    REAL PART OF ZEROS
C        YMZ(I)      8F10.3    IMAG. PART OF ZEROS
C         RR1,RR2, AAG, W, V    5F10.3  RR1 AND RR2 DESIRED RADIUS FOR
C        GRAPH, AAG IS ANGLE IN RADIANS LOCATING POSITION OF
C         COMPENSATING ZERO FOR RADIUS VS ANGLE OF EMERGENCE GRAPH
C         W SUPPRESSES PRINTOUT AS FOLLWS (W = 0.0 PRINTS ONLY EIGHT
C         VALUES, W = 1.0 PRINTS OUT SIXTEEN VALUES FOR EACH RADIUS)
C        V SUPPRESSES GRAPHS AS FOLLOWS ( V = 0.0 ALL GRAPHS
C        SUPPRESSED,  V = 1.0 A TOTAL OF THREE GRAPHS PER RUN
      DIMENSION REP(8),YMP(8),REZ(8),YMZ(8),R(900),B(900),REZ1(900)    ,
     1ZANG(900),ANG(900),ITITLE(12),YMZ1(900),ZA(8),PA(8),GNA(900),
     2 D(900), AZ(900), CEN(900), ACE(900)
      READ 63, NOS
   63 FORMAT(I2)
      DO 402 K= 1,NOS
      READ 2,M,N
    2 FORMAT (2I2)
      READ 4, (REP(I),I = 1,M)
      READ 4, (YMP(I),I= 1,M)
      IF(N-1) 4,9,9
    9 READ 4, (REZ(I),I=1,N)
      READ 4, (YMZ(I), I= 1,N)
    4 FORMAT (8F10.3)
      READ 44, RR1, RR2, AAG,W,V
   44 FORMAT (5F10.3)
C     PRINT DATA READ IN
      PRINT 62,K
```

```
   62 FORMAT (/4X,7HRUN NO.,I2)
      PRINT 6,M,N
    6 FORMAT(//9X,2HM=,I3,20X,2HN=,I3)
      PRINT 8
    8 FORMAT (/5X,7H REP(I),9X,7H YMP(I),9X,7H REZ(I),9X,7H YMZ(I))
      IF(N-1) 48,49,49
   48 PRINT 99, (REP(I),YMP(I), I= 1,M)
      GO TO 99
   49 PRINT 10, (REP(I),YMP(I),REZ(I),YMZ(I),I=1,N)
      LL = N + 1
      PRINT 99, (REP(I), YMP(I), I=LL,M)
   99 FORMAT (2F15.3)
   10 FORMAT (4F15.3)
      PRINT 45, RR1,RR2,AAG,W,V
   45 FORMAT (/4X,5HRR1 =,F10.3, 5X,5HRR2 =,F10.3, 5X,5HAAG =,F10.3,
     15X, 3HW =,F10.3,5X,3HV =,F10.3)
      XX = M-N
      X = XX - 2.
C     ASYMPTOTE VECTOR WITH COMPENSATING ZEROS, THETA
      IF (X) 202,202,203
  202 PRINT 205
  205 FORMAT(/2X,15HTHETA= INFINITY)
      GO TO 206
  203 THETA = 180./X
      PRINT 204, THETA
  204 FORMAT (/2X,6HTHETA=, F10.3)
C     ASYPTOTE VECTOR WITHOUT COMPENSATING ZEROS, ZETA
  206 IF (M-N) 210, 210, 211
  210 PRINT 212
  212 FORMAT (/2X,14HZETA= INFINITY)
      GO TO 240
  211 ZETA = 180./XX
      PRINT 214, ZETA
  214 FORMAT (/2X,5HZETA=,F10.3)
C     ASYMPTOTE CENTROID, CEN = (SUM -POLES - SUM ZEROS)/ M-N
  240 SUM=0.0
      DO 201 I=1,8
  201 SUM=SUM + REP(I)-REZ(I)
      SMM=-SUM
C     ANGLES DUE TO NON-COMP. ZEROS, ANGZ
      ANGZ=0.0
      IF (N) 250,250, 251
  251 DO 12 I = 1,N
      IF(YMZ(I)-YMP(1))71,72,73
   71 IF(REZ(I)-REP(1))74,76,77
   74 ZA(I)=ATANF((YMP(1)-YMZ(I))/(REP(1)-REZ(I)))+3.14159
      GO TO 12
   72 IF(REZ(I)-REP(1))75,80,80
   75 ZA(I)=3.14159
      GO TO 12
   76 ZA(I)=3./2.*3.14159
      GO TO 12
   77 ZA(I)=2.*3.14159-ATANF((YMP(1)-YMZ(I))/(REZ(I)-REP(1)))
      GO TO 12
   80 ZA(I)=0.0
      GO TO 12
```

```
   73 IF(REZ(I)-REP(1))81,82,83
   81 ZA(I)=ATANF((YMZ(I)-YMP(1))/(REZ(I)-REP(1)))
      GO TO 12
   82 ZA(I)=3.14159/2.
      GO TO 12
   83 ZA(I) = ATANF((YMZ(I)-YMP(1))/(REZ(I) - REP(1)))
   12 ANGZ=ANGZ+ZA(I)
C     ANGLES DUE TO POLES, ANGP
  250 ANPG = 0.0
      IF (M-2) 254, 254, 253
  253 DO 14 I = 3,M
      IF(YMP(I)-YMP(1))91,92,93
   91 IF(REP(I)-REP(1))94,96,97
   94 PA(I)=ATANF((YMP(1)-YMP(I))/(REP(1)-REP(I)))+3.14159
      GO TO 14
   92 IF(REP(I)-REP(1))95,100,100
   95 PA(I)=3.14159
      GO TO 14
   96 PA(I)=3./2.*3.14159
      GO TO 14
   97 PA(I)=2.*3.14159-ATANF((YMP(1)-YMP(I))/(REP(I)-REP(1)))
      GO TO 14
  100 PA(I)=0.0
      GO TO 14
   93 IF(REP(I)-REP(1))101,102,103
  101 PA(I) = 3.14159 - ATANF((YMP(I)-YMP(1))/(REP(1) - REP(I)))
      GO TO 14
  102 PA(I)=3.14159/2.
      GO TO 14
  103 PA(I) = ATANF((YMP(I) -YMP(1))/(REP(I)-REP(1)))
   14 ANPG = ANPG + PA(I)
      ANGP = ANPG + 3.14159/2.
      GO TO 252
  254 ANGP = 3.14159/2.
C     ANGLE OF EMERG. IF COMPLEX ZEROS NOT PRESENT, ANGLE
  252 ANGLE=180./3.14159*(-3.14159+ANGP-ANGZ)
C     GENERATE COMPENSATION ZEROS
C     ALT. STATE. 15-19 FOR VARIABLE RADIUS
      DO 301 J=1,20
      CA = J
      R(J)=CA/10.
   15 DO 26 I=1,630
   16 AI=I-1
   17 B(I)=AI/100.
   18 REZ1(I)= REP(1) - COSF(B(I))*R(J)
   19 YMZ1(I) = YMP(1) + SINF(B(I))*R(J)
C     ANGLES DUE TO COMPENSATION ZEROS, ZANG(I)
   53 IF(B(I)-3.14159) 54,54,55
   54 D(I)=180. - B(I)*180./3.14159
      GO TO 56
   55 D(I)=540.-B(I)*180./3.14159
   56 IF(REZ1(I)-REP(1))57,58,59
   57 ZANG(I)=180./3.14159*(3.14159-ATANF((-YMZ1(I)-YMP(1))/
     1(REP(1)-REZ1(I))))+D(I)
      GO TO 20
   58 ZANG(I)=90.+D(I)
```

```
      GO TO 20
   59 ZANG(I)=180./3.14159*(ATANF((-YMZ1(I)-YMP(1))/(REZ1(I)-REP(1))))
     1 + D(I)
C     ANGLE OF EMERGENCE WITH COMPENSATION, ANG(I)
   20 GNA(I)=ANGLE-ZANG(I)
      IF (X) 229,229,230
  230 CEN(I) = (SMM +2. * REZ1(I) )/X
  229 AZ(I) = 180. / 3.14159 * (6.28 - B(I))
   21 IF(GNA(I)) 22,23,23
   22 ANG(I) = -GNA(I)
      IF (ANG(I)) 600,333,333
  600 ANG(I) = 360. + ANG(I)
      GO TO 333
   23 ANG(I) = 360. - GNA(I)
  333 IF (B(I) -AAG) 26,46,26
   46 ACE(J)= ANG(I)
   26 CONTINUE
      IF(R(J)-RR1) 321,27,320
  320 IF(R(J)-RR2)321,27,321
   27 LA=4HANG
      IF(V) 31,29,31
   29 GO TO 321
   31 DO 1 I=1,12
    1 ITITLE(I)=8H
      ITITLE(1)=8H REFS
      ITITLE(2)=8HE.G.
      ITITLE(3)=8HANGLE
      ITITLE(4)=8H OF
      ITITLE(5)=8H EMERG
      ITITLE(6)= 8H VS ZERO
      ITITLE(7) = 8H POSIT
      IF(R(J)-RR1) 330,331,330
  331 ITITLE(8) = 8H R=.5
      GO TO 332
  330 ITITLE(8) = 8H R=1.0
  332 ITITLE(12)= 8H THESIS
      CALL DRAW (630,AZ,ANG,0,0,LA,ITITLE,0,0,2,1,
     1 0,0, 7,9, 0, LAST)
  321 PRINT 405, R(J)
  405 FORMAT (/2X,8HRADIUS =, F10.3)
  220 PRINT 28
   28 FORMAT (8X,14H ANGLE TO ZERO,5X, 19H ANGLE OF EMERGENCE, 5X,
     1 9H CENTROID)
      IF(W) 60,61,60
   60 PRINT 30, AZ(1),ANG(1), CEN(1)
      PRINT 30, AZ(39)   , ANG(39), CEN(39)
      PRINT 30, AZ(78) , ANG (78) , CEN(78)
      PRINT 30, AZ(117), ANG(117), CEN(117)
      PRINT 30, AZ(157),  ANG (157), CEN (157)
      PRINT 30, AZ(196), ANG(196), CEN(196)
      PRINT 30, AZ(235), ANG(235), CEN(235)
      PRINT 30, AZ(276), ANG(276), CEN(276)
      PRINT 30, AZ(314), ANG(314), CEN(314)
      PRINT 30, AZ(353), ANG(353), CEN(353)
      PRINT 30, AZ (392), ANG(392), CEN(392)
      PRINT 30, AZ(431), ANG(431), CEN(431)
```

```
      PRINT 30, AZ(471), ANG(471), CEN(471)
      PRINT 30, AZ(510), ANG(510), CEN(510)
      PRINT 30, AZ(549), ANG (549), CEN(549)
      PRINT 30, AZ(588), ANG(588), CEN(588)
      GO TO 30
   61 PRINT 30, AZ(1),ANG(1), CEN(1)
      PRINT 30, AZ(78) , ANG (78) , CEN(78)
      PRINT 30, AZ(157),  ANG (157), CEN (157)
      PRINT 30, AZ(235), ANG(235), CEN(235)
      PRINT 30, AZ(314), ANG(314), CEN(314)
      PRINT 30, AZ (392), ANG(392), CEN(392)
      PRINT 30, AZ(471), ANG(471), CEN(471)
      PRINT 30, AZ(549), ANG (549), CEN(549)
   30 FORMAT (3F20.3)
  301 CONTINUE
      PRINT 401, ANGZ, ANGP
  401 FORMAT(/6HANGZ =,F10.3,5X,6HAMGP =,F10.3)
      PRINT 32, ANGLE
   32 FORMAT (/26HANGLE OF EMERG. W/O COMP =,F10.3)
C     IF V = 0.0, NO GRAPH.  IF V = 1.0  A GRAPH OF R VS ANG OF EMERG
      IF(V) 66,65,66
   65 GO TO 402
   66 LA=4HANG
      LA =4HANG
      DO 47 I = 1,12
   47 ITITLE (I)=8H
      ITITLE(1)=8H REEFS
      ITITLE(2)=8HE.G.
      ITITLE(3)= 8HRADIUS
      ITITLE(4)= 8H VS
      ITITLE(5) = 8H ANGLE
      ITITLE(6) = 8HOF EMERG
      CALL DRAW (20,R, ACE,0,0,LA,ITITLE,0,0,2,1,
     10,0,7,9,0,LAST)
  402 CONTINUE
   67 END
      END

1
4
1.0         1.0                   12.0
-6.0        6.0
1.2         2.4         -3.77               1.0
```

```
..JOB0414F, REES,F. G.
      PROGRAM EQN
      READ 240, NUMB
  240 FORMAT (I2)
      DO 241  I = 1, NUMB
  241 CALL SPLANE
      END
      SUBROUTINE SPLANE
      DIMENSION R(129),X(129),IT(10),ROOTR(128),ROOTI(128),ITITLE(12),
     1A(129),B(129),ROOTJ(128),ROOTM(128),AP(129),AZ(129)
      COMMON R,VAR,NO,ROOTR,ROOTI,AP,X
  206 MOD=1
      LAB=4H
   68 FORMAT(8E15.5)
   70 FORMAT    (/,14HIMAGINARY PART,//)
   69 FORMAT    (/,9HREAL PART,//)
  200 FORMAT(6A8)
  203 FORMAT(I3)
  204 FORMAT(E10.5)
      READ 200,(ITITLE(I),I=1,6)
      PRINT 324
  324 FORMAT(1H1,11HGRAPH TITLE,/)
      PRINT 200,(ITITLE(I),I=1,6)
   28 FORMAT   (/,36HORDER OF THE CHARACTERISTIC EQUATION,/)
      PRINT 28
      READ 203,NO
      PRINT 203,NO
      N=NO+1
  205 FORMAT(8E10.5)
  207 FORMAT (8E12.5)
   22 FORMAT   (/,53HCONSTANT PART OF THE COEFFICIENTS IN DESCENDING ORDE
     1R,/)
      PRINT 22
      READ 205,(A(K),K=1,N)
      PRINT 207,(A(K),K=1,N)
   23 FORMAT   (/,53HVARIABLE PART OF THE COEFFICIENTS IN DESCENDING ORDE
     1R,/)
      PRINT 23
      READ 205,(B(K),K=1,N)
      PRINT 207,(B(K),K=1,N)
      VAR = .001
      ND = 8
      XSCALE = 2.0
      YSCALE=XSCALE
  201 FORMAT (/////,21H0THE SYSTEM POLES ARE,/)
      PRINT 201
      M=N
      DO 67 K=1,N
      AP(K)=A(M)
   67 M=M-1
      DO 432 K=1,30
      ROOTI(K)=0.
  432 ROOTR(K)=0.
      NX=NO
      DO 15 K=1,31
   15 X(K)=0.0
```

```
      CALL POLYRT (AP,X,NX,ROOTR,ROOTI,1.E-5)
      PRINT 69
      PRINT 68,(ROOTR(K),K=1,NO)
      PRINT 70
      PRINT 68,(ROOTI(K),K=1,NO)
      CALL DRAW(NO,ROOTR,ROOTI,MOD,1,LAB,ITITLE,XSCALE,YSCALE,
     11,7,2,2,8,9,0,LAST)
      MOD=2
  202 FORMAT (//////,21H THE SYSTEM ZEROS ARE,/)
      K=1
    3 IF(B(K)) 1,2,1
    2 K=K+1
      GO TO 3
    1 NORD=N-K
      IF(NORD-1) 6,4,5
    4 ZERO=-B(K+1)/B(K)
    7 FORMAT (/,16HTHE SYSTEM ZERO=,E15.5,/)
      PRINT 7,ZERO
      ROOTM(1)=ZERO
      ROOTM(2)=16.*XSCALE
      ROOTJ(1)=0.
      ROOTJ(2)=0.
      NORD=2
      GO TO 11
    6 PRINT 9
    9 FORMAT (/,25HALL ZEROS ARE AT INFINITY)
      GO TO 8
    5 NN=NORD+1
      DO 10 L=1,NN
      R(L)=B(K)
   10 K=K+1
      PRINT 202
      M=NN
      DO 46 K=1,NN
      AZ(K)=R(M)
   46 M=M-1
      DO 433 K=1,30
      ROOTM(K)=0.
  433 ROOTJ(K)=0.
      NORX=NORD
      DO 16 K=1,31
   16 X(K)=0.0
      CALL POLYRT(AZ,X,NORX,ROOTM,ROOTJ,1.E-5)
      PRINT 69
      PRINT 68,(ROOTM(K),K=1,NORD)
      PRINT 70
      PRINT 68,(ROOTJ(K),K=1,NORD)
   11 CALL DRAW(NORD,ROOTM,ROOTJ,MOD,3,LAB,ITITLE,XSCALE,YSCALE,
     11,7,2,2,8,9,0,LAST)
    8 CONTINUE
      GO TO(31,32,33,34,35,36,37,38,39,40),ND
   31 G=1.0076
      GO TO 41
   32 G=1.016
      GO TO 41
   33 G=1.0248
```

```
      GO TO 41
   34 G=1.0312
      GO TO 41
   35 G=1.0394
      GO TO 41
   36 G=1.0483
      GO TO 41
   37 G=1.0568
      GO TO 41
   38 G=1.0633
      GO TO 41
   39 G=1.071
      GO TO 41
   40 G=1.078
   41 CONTINUE
      DO 101 J=1,30
      DO 100 K=1,10
      DO 300 L=1,N
  300 R(L)=A(L)+B(L)*VAR
      CALL ROOTX
      DO 71 JJ=1,NO
      IF(ABSF(ROOTI(JJ))-5.E-04) 61,61,62
   61 ROOTJ(JJ)=0.
      GO TO 71
   62 ROOTJ(JJ)=ROOTI(JJ)
   71 CONTINUE
  100 VAR=VAR*G
      IF(J-30) 101,98,98
   98 MOD=3
  101 CALL DRAW(NO,ROOTR,ROOTJ,MOD,2,LAB,ITITLE,XSCALE,YSCALE,
     11,7,2,2,8,9,0,LAST)
      GO TO 206
      END
      SUBROUTINE ROOTX
      DIMENSION C(31),D(29),R(129),ROOTR(128),ROOTI(128),EE(31),FE(31)
     1,AP(129),X(129)
      COMMON R,VAR,NO,ROOTR,ROOTI,AP,X
      M=1
      DO 1492 MA=1,31
      EE(MA)=0.
 1492 FE(MA)=0.
   20 BETAN=ROOTI(M)+EE(M)
      ALFAN=ROOTR(M)+FE(M)
      DO 7 I=1,100
      S=2.*ALFAN
      T=-(ALFAN**2+BETAN**2)
      C(1)=R(1)
      C(2)=R(2)+S*R(1)
      NC=NO+1
      DO 2 L=3,NC
    2 C(L) = R(L)+S*C(L-1) + T*C(L-2)
      AN= C(NO+1)-ALFAN*C(NO)
      BN= BETAN*C(NO)
      IF (NO-3) 21, 17, 18
   17 CN = 3.*R(1)*(ALFAN**2-BETAN**2) + 2.*R(2)*ALFAN + R(3)
      DN = 6.*R(1)*ALFAN*BETAN + 2.*R(2)*BETAN
```

```
      GO TO 19
   21 CN = 2.*R(1)*ALFAN + R(2)
      DN  = 2.*R(1)*BETAN
      GO TO 19
   18 D(1) = C(1)
      D(2)=C(2)+S*D(1)
      NU=NO-1
      DO 3 N=3,NU
    3 D(N)=C(N)+S*D(N-1)+T*D(N-2)
      CN= C(NO)-2.*D(NO-2)*BETAN**2
      DN= 2.*BETAN*(D(NO-1)-ALFAN*D(NO-2))
   19 ALFA=ALFAN-(AN*CN+BN*DN)/(CN**2+DN**2)
      BETA =BETAN + (AN*DN-BN*CN)/(CN**2+DN**2)
      AZETA=.01+ABSF(BETAN)
      ZETA=SQRTF(.01-T)
      IF(ABSF((ALFA-ALFAN)/ZETA )-.1)55,55,52
   55    IF(ABSF((BETA-BETAN)/AZETA)-1.)56,56,52
   56 EE(M) =(ALFA-ALFAN)*10. + BETA - ROOTI(M)
      FE(M)=(BETA - BETAN)*10. +ALFA - ROOTR(M)
      IF(ABSF((ALFA-ALFAN)/(ALFAN+1.))-5.E-4)4,4,5
    4 IF(ABSF((BETA-BETAN)/(BETAN+1.))-5.E-4)6,6,5
    5 ALFAN=ALFA
    7 BETAN=BETA
   52 CONTINUE
      MR=NC
      DO 6777 K=1,NC
      AP(K)=R(MR)
 6777 MR=MR-1
      DO 434 K=1,30
      ROOTI(K)=0.
  434 ROOTR(K)=0.
      NX=NO
      DO 83 K=1,31
   83 X(K)=0.0
      CALL POLYRT (AP,X,NX,ROOTR,ROOTI,1.E-5)
      GO TO 16
    6 ROOTR(M)=ALFA
    1 ROOTI(M)=BETA
   12 IF(M-NO) 15,16,16
   15 M=M+1
      GO TO 20
   16 RETURN
      END
      END

 2
REES, E.G.   THESIS
  5
 1.0        19.0      131.      513.      1488.     1088.
                                1.0       3.0       18.25
  REES, E. G.  THESIS
  6
 1.0         24.      221.      978.      2888.      6528.
                                          1.0        1.0       16.25
```

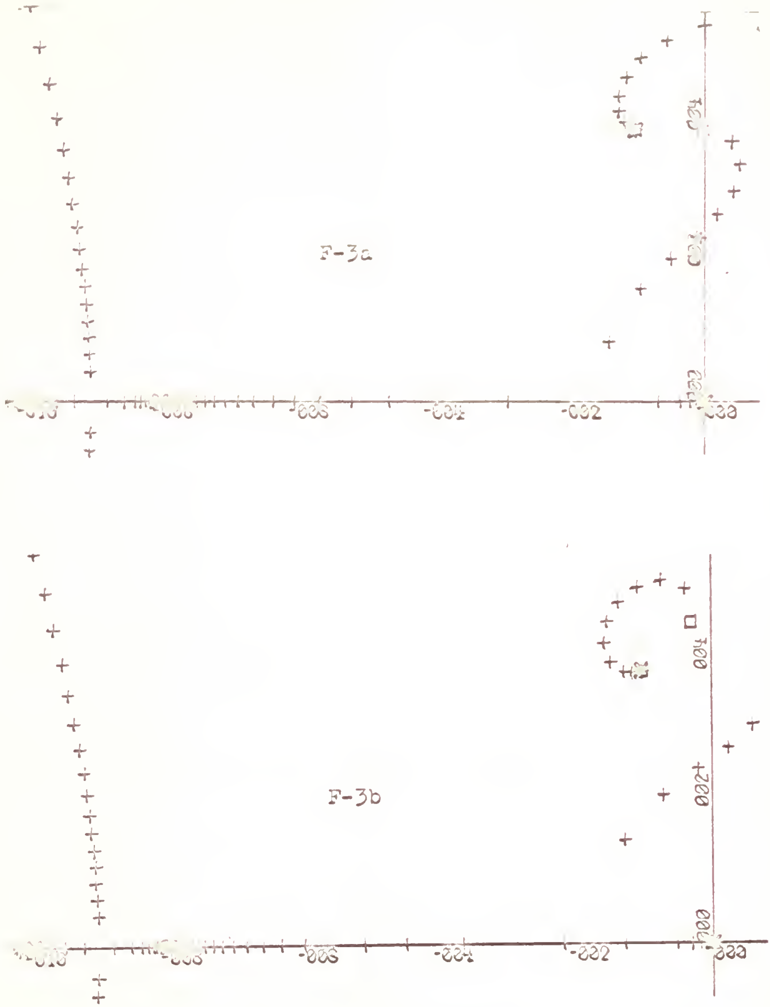


Fig. 11 Root Locus Data (Computer)

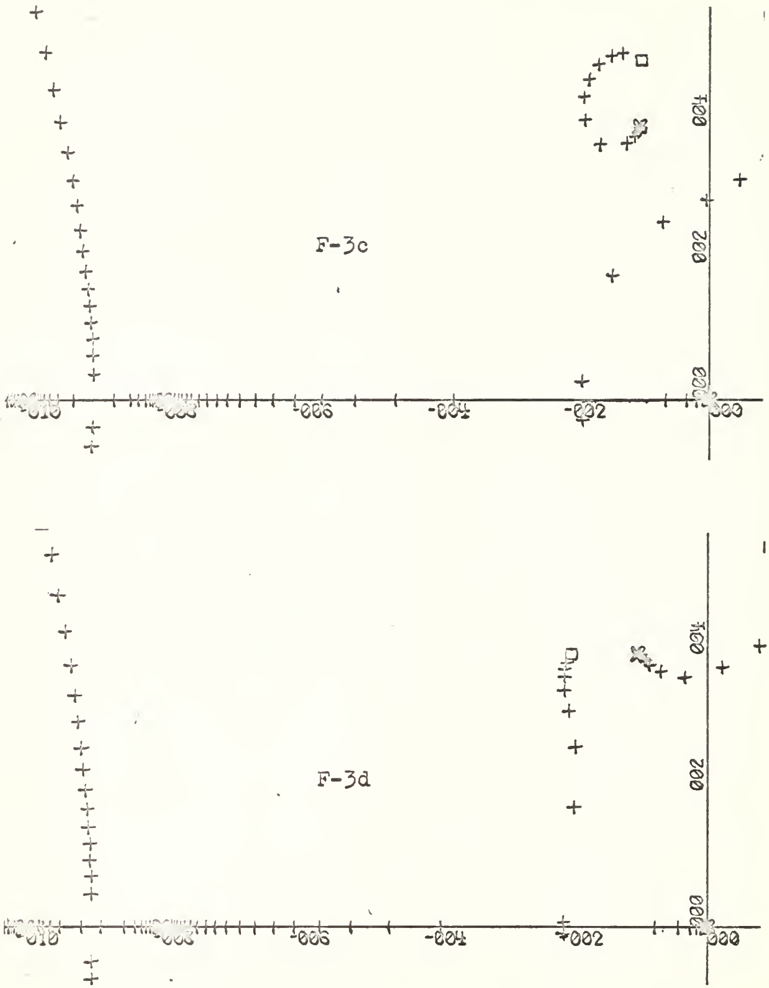
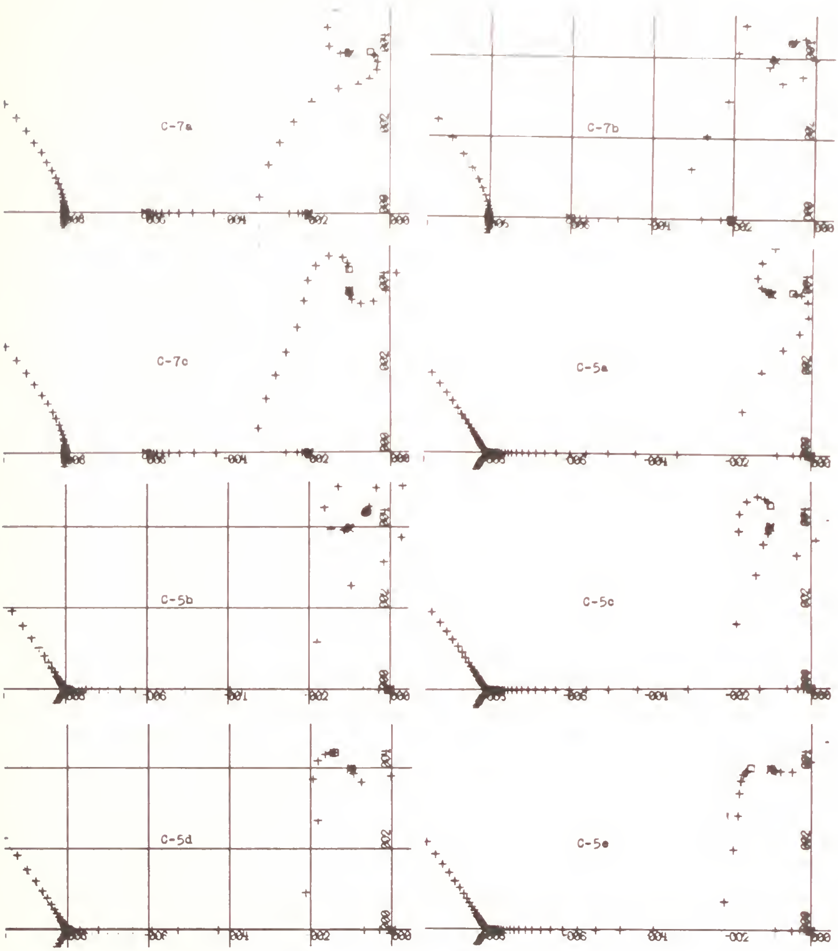


Fig. 12 Root Locus Data (Computer)



Fig

TABLE 2

CHARACTERISTIC EQUATIONS

Desig	θ	s^6	s^5	s^4	s^3	s^2	s^1	s^0
A-1	*		1	18	113	400	1088	0
A-1a	000					1	1	16.25
A-1b	045					1	1.2	19.56
A-1c	090					1	2	21.25
A-1d	135					1	2.8	21.16
A-2	*		1	10	49	168	272	0
A-2a	000					1	1	16.25
A-2b	045					1	1.2	19.56
A-2c	090					1	2	21.25
A-2d	135					1	2.8	21.16
A-2e	180					1	3	18.25
A-2f	225					1	2.8	15.06
A-2g	315					1	1.2	13.46
A-3	*		1	19	131	513	1488	1088
A-3a	000					1	1	16.25
A-3b	090					1	2	21.25
A-3c	180					1	3	18.25
A-3d	270					1	2	13.25
A-4	*		1	20	149	626	1888	2176
A-4a	000					1	1	16.25
A-4b	045					1	1.2	19.56
A-4c	090					1	2	21.25
A-4d	135					1	2.8	21.16
A-4e	180					1	3	18.25
A-4f	225					1	2.8	15.06
A-4g	270					1	2	13.25
A-4h	315					1	1.2	13.46
A-5	*		1	18	113	400	1088	0
A-5a						1	1.5	16.56
A-5b						1	1.62	18.26
A-5c						1	2.5	17.56
A-5d						1	2.38	19.01
A-5e						1	2	19.1
A-5f						1	2	15.05
A-5g						1	1.62	15.2
A-6	*		1	18	113	400	1088	0
A-6a	000					1	2	16.01
A-6b	045					1	6	22.19
A-6c	090					1	2	26.0
A-6d	135					1	34	24.97
A-6e	180					1	2	10.0

Desig	θ	s^6	s^5	s^4	s^3	s^2	s^1	s^0
B-1	*		1	18	113	400	1088	0
B-1a	000				1	2	17.25	16.25
B-1b	045				1	2.2	20.75	19.56
B-1c	090				1	3.0	23.25	21.25
B-1d	180				1	4.0	21.25	18.25
B-1e	270				1	3.0	15.25	13.25
B-2	*		1	18	113	400	1088	0
B-2a	000				1	8	23.25	113.75
B-2b	045				1	8.2	27.96	136.92
B-2e	090				1	9.0	35.25	148.75
B-2d	180				1	10	39.25	127.75
B-2e	270				1	9	27.25	92.75
B-3	*		1	18	113	400	1088	0
B-3a	000				1	10	25.25	146.25
B-3b	045				1	10.2	20.36	176.04
B-3c	090				1	11.0	39.25	191.25
B-3d	135				1	11.8	46.36	190.44
B-3e	180				1	12.0	45.25	164.25
B-3f	270				1	11.0	31.25	119.25
B-3g	315				1	10.2	24.26	120.94
B-4	*		1	10	49	168	272	0
B-4a	000				1	2.0	17.25	16.25
B-4b	090				1	2.2	20.75	19.56
B-4c	180				1	3.0	23.25	21.25
B-4d	270				1	4.0	21.25	18.25
B-5	*		1	10	49	168	272	0
B-5a	000				1.0	4.0	19.25	48.75
B-5b	045				1.0	4.2	23.16	58.68
B-5c	090				1.0	5.0	27.25	63.75
B-5d	225				1.0	6.0	24.25	54.75
B-5e	270				1.0	5.0	19.25	39.75
B-6	*		1	10	49	168	272	0
B-6a	000				1	6.0	21.25	81.25
B-6b	045				1	6.2	25.56	97.80
B-6c	090				1	7.0	31.25	106.25
B-6d	180				1	8.0	33.25	91.25
B-6e	270				1	7.0	23.25	66.25
C-1	*	1.0	10.5	122	456.5	1288	544	0
C-1a	000					1	1	16.25
C-1b	045					1	1.2	19.56
C-1c	090					1	2	21.25

Table 2 (Cont'd)

Desig	θ	s^6	s^5	s^4	s^3	s^2	s^1	s^0
C-1d	135					1	2.8	21.16
C-1e	180					1	3.0	18.25
C-1f	225					1	2.8	15.06
C-1g	270					1	2.0	13.25
C-1h	315					1	1.2	13.46
C-2	*	1	19	131	513	1488	1088	0
C-2a	000					1	1.0	16.25
C-2b	045					1	1.2	19.56
C-2c	090					1	2.0	21.25
C-2d	135					1	2.8	21.16
C-2e	180					1	3.0	18.25
C-2f	225					1	2.8	15.06
C-2g	270					1	3.0	18.25
C-2h	315					1	2	15.05
C-3	*	1	20	149	626	1888	2176	0
C-3a	000					1	1	16.25
C-3b	045					1	1.2	19.56
C-3c	090					1	2	21.25
C-3d	135					1	2.8	21.16
C-3e	180					1	3.0	18.25
C-3f	225					1	2.8	15.06
C-3g	270					1	2.	13.25
C-3h	315					1	1.2	13.46
C-4	*	1	24	221	1078	3480	6528	0
C-4a	000					1	1	16.25
C-4b	045					1	1.2	19.56
C-4c	090					1	2	21.25
C-4d	135					1	2.8	21.16
C-4e	225					1	2.8	15.06
C-4f	315					1	1.2	13.46
C-5	*	1	26	257	1304	4288	8704	0
C-5a	000					1	1	16.25
C-5b	045					1	1.2	19.56
C-5c	090					1	2	21.25
C-5d	135					1	2.8	21.16
C-5e	180					1	3.0	18.25
C-5f	225					1	2.8	15.06
C-5g	270					1	2	13.25
C-5h	315					1	1.2	13.46
C-6	*	1	25	245	1299	4566	10016	0
C-6a	000					1	1	16.25
C-6b	045					1	1.2	19.56

Table 2 (Cont'd)

Desig	θ	s^6	s^5	s^4	s^3	s^2	s^1	s^0
C-6c	090					1	2	21.25
C-6d	135					1	2.8	21.16
C-6e	225					1	2.8	15.06
C-7	*	1	26	269	1520	5644	13504	13056
C-7a	000					1	1	16.25
C-7b	045					1	1.2	19.56
C-7c	090					1.	2	21.25
C-8	*	1	34	465	3360	14720	43008	69632
C-8a	000					1	1	16.25
C-8b	045					1	1.2	19.56
C-8c	090					1	2	21.25
C-8d	135					1	2.8	21.16
C-8e	225					1	2.8	15.06
C-8f	270					1	2	13.25
C-8g	315					1	1.2	13.46
D-1	*	1	24	229	1154	2168	3200	0
D-1a	000					1	5	22.25
D-1b	045					1	5.2	25.94
D-1c	090					1	6.0	29.25
D-1d	135					1	6.8	30.74
D-1e	225					1	6.7	24.5
D-1f	315					1	5.3	20.54
D-2	*	1	25	251	1339	3952	4800	0
D-2a	000					1	5.0	22.25
D-2b	045					1	5.2	25.94
D-2c	135					1	6.8	20.74
D-2d	225					1	6.7	24.5
D-2e	315					1	5.3	20.54
D-3	*	1	30	373	2528	10092	22208	19200
D-3a	000					1	5	22.25
D-3b	045					1	5.2	25.94
D-3c	090					1	6.0	29.25
D-3d	135					1	6.8	30.74
D-3e	225					1	6.7	24.5
D-3f	315					1	5.3	20.54
D-4	*	1	32	405	2634	9440	16000	00
D-4a	000					1	5	22.25
D-4b	045					1	5.2	25.94
D-4c	090					1	6.0	29.25
D-4d	135					1	6.8	30.74
D-4e	180					1	7.0	28.25

Table 2 (Cont'd)

Desig	θ	s^6	s^5	s^4	s^3	s^2	s^1	s^0
D-4f	225					1	6.7	24.5
D-4g	270					1	6.0	21.25
D-4h	315					1	5.3	20.54
D-5	*	1	30	373	2528	10092	22208	19200
D-5a	000					1	5	22.25
D-5b	045					1	5.2	25.94
D-5c	090					1	6.0	29.25
D-5d	135					1	6.8	30.74
D-5e	225					1	6.7	24.5
D-5f	315					1	5.3	20.54
D-6	*	1	34	469	3444	14708	34880	32000
D-6a	000					1	5	22.25
D-6b	045					1	5.2	25.94
D-6c	090					1	6.0	29.25
D-6d	135					1	6.8	30.74
D-6e	225					1	6.7	24.5
D-6f	315					1	5.3	20.54
E-1	*	1	24	221	1078	3488	6528	0
E-1a	000				1	4	1925	48.75
E-1b	045				1	4.2	23.16	58.68
E-1c	090				1	5.0	27.25	63.75
E-1d	225				1	6.0	24.25	54.75
E-1e	270				1	5.0	19.25	39.75
E-2	*	1	24	221	1078	3488	6528	0
E-2a	000				1	6	21.25	81.25
E-2b	045				1	6.2	25.56	97.8
E-2c	090				1	7.0	31.25	106.25
E-2d	180				1	8.0	33.25	91.25
E-2e	270				1	7.0	23.25	66.25
E-3	*	1	24	221	1078	3488	6528	0
E-3a	000				1	8	23.25	113.75
E-3b	045				1	8.2	27.96	136.92
E-3c	090				1	9.0	35.25	148.75
E-3d	180				1	10.0	39.25	217.75
E-3e	270				1	9	27.25	92.75
E-4	*	1	24	221	1078	3488	6528	0
E-4a	000				1	10	25.25	146.25
E-4b	045				1	10.2	30.36	176.04
E-4c	090				1	11.0	39.25	191.25
E-4d	180				1	12.0	45.25	164.25
E-4e	270				1	11.0	31.25	119.25

Desig	0	s^8	s^7	s^6	s^5	s^4	s^3	s^2	s^1	s^0
F-1	*		1	27	293	1741	6722	16992	19584	0
F-1a	000							1	1	16.25
F-1b	045							1	1.2	19.56
F-1c	090							1	2	21.25
F-1d	180							1	.2	16.01
F-1e	225							1	.6	22.19
F-1f	315							1	2	10
F-2	*		1	30	365	2404	9956	27456	39168	0
F-2a	000							1	1.2	19.56
F-2b	045							1	2	21.25
F-2e	090							1	.2	16.01
F-2d	180							1	.6	22.19
F-2e	225							1	2	26.0
F-2f	315							1	2	10
F-3	*		1	38	573	4460	20388	61760	108800	0
F-3a	000							1	.2	16.01
F-3b	045							1	1.62	18.26
F-3c	090							1	2	26
F-3d	180							1	2	10
F-3e	315							1	.6	10.98
F-4	*		1	28	320	2034	8463	23714	36576	19584
F-4a	000							1	1	16.25
F-4b	045							1	1.2	19.56
F-4c	090							1	2.8	21.16
F-4d	225							1	2.8	15.06
F-4e	315							1	1.2	13.46
G-1	*		1	27	293	1741	6722	16992	19584	0
G-1a	000						1	2	17.25	16.25
G-1b	045						1	2.2	20.75	19.56
G-1c	090						1	3.0	23.25	21.25
G-1d	180						1	4.0	21.25	18.25
G-1e	270						1	3.0	15.25	13.25
G-2	*		1	27	293	1741	6722	16992	19584	0
G-2a	000						1	6	21.25	81.25
G-2b	045						1	6.2	25.56	97.8
G-2c	090						1	7.0	31.25	106.25
G-2d	180						1	8.0	33.25	91.25
G-2e	270						1	7.0	23.25	66.25
G-3	*		1	27	293	1741	6722	16992	19584	0
G-3a	000						1	8	23.25	113.75
G-3b	045						1	8.2	27.96	136.92
G-3c	090						1	9.0	35.25	148.75
G-3d	180						1	10	39.25	127.75

Desig	θ	s^8	s^7	s^6	s^5	s^4	s^3	s^2	s^1	s^0
G-3e	270						1	9.0	27.25	92.75
G-4	*		1	27	293	1741	6722	16992	19584	0
G-4a	000						1	10	25.25	146.25
G-4b	045						1	10.2	30.36	176.04
G-4c	090						1	11	39.25	191.25
G-4d	180						1	12	45.25	164.25
G-4e	270						1	11	31.25	119.25
H-1	*		1	28	320	2034	8463	23714	36576	19584
H-1a	000						1	1	16.25	0
H-1b	045						1	1.2	19.56	0
H-1c	090						1	2.0	21.25	0
H-1d	180						1	3.0	18.25	0
H-1e	270						1	2.0	13.25	0
H-2	*		1	28	320	2034	8463	23714	36576	0
H-2a	000						1	6.0	21.25	81.25
H-2b	045						1	6.2	25.56	97.8
H-2c	090						1	7.0	31.25	106.25
H-2d	180						1	8.0	33.25	91.25
H-2e	270						1	7.0	23.25	66.25
H-3	*		1	28	320	2034	8463	23714	36576	19584
H-3a	000						1	8	23.25	113.75
H-3b	045						1	8.2	27.96	136.92
H-3c	090						1	9.0	35.25	148.75
H-3d	180						1	10	39.25	127.75
H-3e	270						1	9	27.25	92.75
H-4	*		1	28	320	2034	8463	23714	36576	19584
H-4a	000						1	10	25.25	146.25
H-4b	045						1	10.2	30.36	176.04
H-4c	090						1	11.0	39.25	191.25
H-4d	180						1	12.0	45.25	164.25
H-4e	270						1	11.0	31.25	119.25

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13 ABSTRACT

Simple analytical methods for the determination of the angle of emergence of complex poles and zeros are developed. Several case studies of root loci emerging from complex zeros are analyzed to determine criterion for the loci starting from complex pole to end on complex zero, as a pre-requisite study for "cancel-compensation" and self-adaptive compensators.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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